High School Mathematics Synopsis

Math 114 – Calculus I

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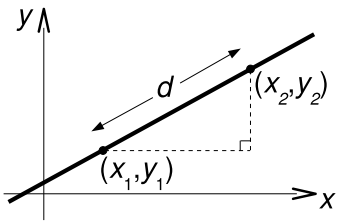
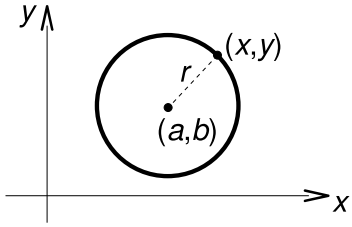
Analytic Geometry





Slope of a line: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

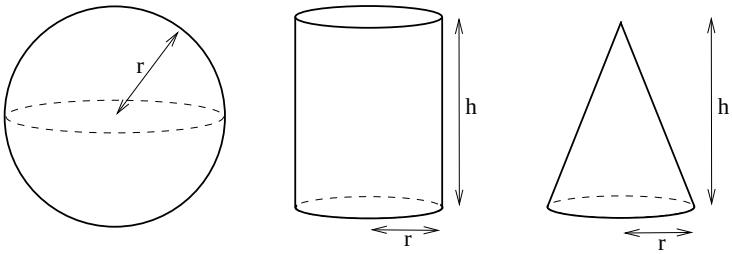
Equation of a line: $y - y_1 = m(x - x_1)$

Distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
(you should be able to derive this from Pythagoras' Theorem)

Circles: equation: $(x - a)^2 + (y - b)^2 = r^2$
area: $A = \pi r^2$
circumference: $C = 2\pi r$

Solid Geometry

	Volume	Surface Area
sphere	$\frac{4}{3}\pi r^3$	$4\pi r^2$
cylinder	$\pi r^2 h$	$2\pi rh + 2\pi r^2$
cone	$\frac{1}{3}\pi r^2 h$	



Classical Algebra

Factorizations:

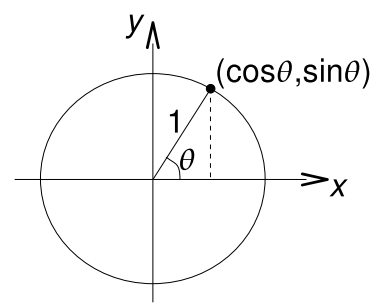
$$b^2 - a^2 = (a+b)(a-b)$$
$$b^3 - a^3 = (b-a)(b^2+ab+a^2)$$
$$b^3 + a^3 = (b+a)(b^2-ab+a^2)$$

Geometric sum: $1 + r + r^2 + \cdots + r^{n-1} = \dfrac{1-r^n}{1-r}$ for $r \neq 1$

Binomial theorem: $(a+b)^n = \displaystyle\sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$ where $\displaystyle\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Trigonometric Functions

Definition:



For any real number θ, the functions $\cos\theta$ and $\sin\theta$ are defined to be the x- and y-coordinates of the point on the unit circle such that the radius makes an angle of θ radians with the positive x-axis, measured counterclockwise.

The tan, sec and csc functions are defined as follows:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

Properties:

Pythagorean identities:

$$\cos^2\theta + \sin^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$

Periodicity:

$$\cos(\theta + 2\pi) = \cos(\theta)$$
$$\sin(\theta + 2\pi) = \sin(\theta)$$
$$\tan(\theta + \pi) = \tan(\theta)$$

Symmetry

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

Addition identities:

$$\cos(x+y) = \cos x\cos y - \sin x\sin y$$
$$\sin(x+y) = \sin x\cos y + \cos x\sin y$$
$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$$

Double angle identities:

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$

Exponential and Logarithmic Functions

Definition of a Logarithm: $y = \log_a x$ means that $x = a^y$ for all $x > 0$.

Cancellation identities:
$\log_a(a^y) = y$ for all y
$a^{\log_a x} = x$ for all $x > 0$

Basic properties:
$a^p a^q = a^{p+q}$
$(a^p)^q = a^{pq}$
$a^p b^p = (ab)^p$
$a^{p/q} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p$
$\log_a(xy) = \log_a x + \log_a y$
$\log_a(\frac{x}{y}) = \log_a x - \log_a y$
$\log_a(x^y) = y \log_a x$

Change of base:
$b^x = a^{x \log_a b}$
$$\log_b x = \frac{\log_a x}{\log_a b}$$

Very common error: note that $\sqrt[q]{x+y} \neq \sqrt[q]{x} + \sqrt[q]{y}$

The Absolute Value

Definition:
$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Properties:
$|ab| = |a||b|$
$|a + b| \leq |a| + |b|$
$|a| < b$ is equivalent to $-b < a < b$