# Asymptotic Throughput Capacity Analysis of VANETs Exploiting Mobility Diversity 

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#### Abstract

Vehicular ad hoc networks (VANETs) rely on intervehicle relay to extend the communication range of individual vehicles for message transmissions to road side units (RSUs). With the presence of a large number of fast-motion vehicles in the network, the end-to-end transmission performance from individual vehicles to RSUs through inter-vehicle relaying is, however, highly unreliable due to the violative inter-vehicle connectivity. As an effort towards this issue, this paper develops an efficient message routing scheme which can maximize the message delivery throughput from vehicles to RSUs. Specifically, we first develop a mathematical framework to analyze the asymptotic throughput scaling of VANETs. We demonstrate that in urban-like layout, the achievable uplink throughput per vehicle from vehicle to RSUs scales as $\Theta\left(\frac{1}{\log n}\right)$ when the number of RSUs scales as $\Theta\left(\frac{n}{\log n}\right)$ with $n$ denoting vehicle population. By noting that the network throughput is bottlenecked by the unbalanced data traffic generated by hotspots of realistic urban areas which may overload the RSUs nearby, a novel packet forwarding scheme is proposed to approach the optimal network throughput by exploiting the mobility diversity of vehicles to balance the data traffic across the network. Using extensive simulations based on realistic traffic traces, we demonstrate that the proposed scheme can improve the network throughput approaching the asymptotic throughput capacity.


Index Terms - Vehicular ad hoc network, asymptotic throughput capacity, packet forwarding scheme, mobility diversity.

## I. Introduction

Vehicular ad hoc networks (VANETs) have recently emerged as a promising technology for providing revolutionized broadband services to vehicles. By deploying wireless gateways (e.g., road side units (RSUs)) along highways/sidewalks and equipping vehicles with on-board communication facilities (e.g., on-board units (OBUs)), two communication modes are enabled for vehicles on the move: vehicle-to-RSU (V2R) com-

[^0]munications and vehicle-to-vehicle (V2V) communications, alternatively known as vehicle-to-infrastructure communications and inter-vehicle communications, respectively. In this framework, three different categories of applications can be supported in general, i.e., road safety applications (e.g., incident warning, traffic alerts), infotainment delivery (e.g., video streaming, online gaming), and the traffic monitoring/management [1] [2]. Motivated by the significant commercial potentials, prominent industrial corporations have also launched multiple projects to promote vehicular communications. For example, "Toyota Friend" builds a private social network for the owners of Toyota cars [3].

In general, most VANET applications (e.g., vehicular video conferencing and traffic monitoring) rely on connections to remote Internet servers through RSUs. To extend the limited communication range of vehicle-to-RSU communications, thus inter-vehicle relaying is typically used with V2V communications. For example, considering the uplink scenario of VANETs ${ }^{1}$, vehicles help each other to relay data towards RSUs, which then forward received data to the remote server via wired networks [4]. However, due to the fast mobility of vehicles and dynamic topologies, the transient and intermittent connections among vehicles make inter-vehicle transmission performance highly unreliable. As a result, enabling quality-of-service ( QoS ) guaranteed transmission from vehicle to an RSU and further to Internet servers for different applications becomes a challenging task. To provide effective and efficient VANET applications, extensive research has been undertaken in different networking layers to address the existing challenges. [5] proposes a transmission control protocol that adapts communication rate and power based on the dynamics of a vehicular network and safety-driven tracking process. [6] studies the routing performance for broadcast-based safety applications in VANETs, and demonstrates that the deployment of RSUs can improve network connectivity. [7] develops a mathematical model to evaluate the average download delay of mobile users, and formulates the content replication problem in roadside unit buffers as a stochastic programming problem.

On the other hand, as the quality of applications keenly relies on the number of vehicles contending for transmissions and the availability of RSUs, the investigation on how nodal throughput scales with the number of vehicles and the

[^1]availability of RSUs in VANETs (i.e., asymptotic network throughput capacity) is crucial in adopting the appropriate network mechanisms (e.g., signaling exchanging) and guiding the real-world network planning (e.g., RSU deployment). The studies on the capacity scaling law of VANETs can date back to the extensively investigated context of mobile ad hoc networks (MANETs) [8], [9]. However, different from MANETs, VANETs typically involves a great network population and high nodal mobility conformed to street layout, and network connectivity can be enhanced by stationary infrastructure (RSUs). Therefore, the existing works on MANETs cannot be directly applied to evaluate the capacity scaling of VANETs.

The asymptotic throughput capacity of VANETs has been studied in a collections of research works [10]-[12], however, with certain spatial limitations on vehicle's mobility. For example, in [10], [11], each vehicle moves on a single road section. In [12], vehicles move on multiple roads but within the predefined Manhattan grid with restricted mobility, i.e., vehicles are mobile around their own center spots with powerlaw distribution. Therefore, by considering a more general scenario, in which vehicles can move across the whole area along roads without above spatial mobility restriction, our work is devoted to characterizing a more generic scaling law of achievable throughput capacity with the RSU deployment in the network. In addition, referred to the recent works on RSU deployment in the urban area, most of the existing works, e.g., [13] and [14], focus on selecting the optimal locations for either RSUs or Access Points, based on a given candidate location set. However, the asymptotic bound on the number of RSUs in the network, i.e., the scaling law of RSU deployment, and its performance on the throughput capacity have rarely been studied before.

To address above issues, in this paper, we develop a generic analytical framework to characterize the capacity scaling law of hybrid VANET. In particular, we address the following three issues:
(Q1): What is the asymptotic throughput capacity of VANETs in uplink scenario?
(Q2): How to optimally determine the scalability of RSUs to achieve the asymptotic throughput capacity?
(Q3): How to improve throughput performance in reality to approach the theoretical throughput capacity?
Specifically, considering the unique dynamic features of VANETs, $Q 1$ is first addressed to derive the scaling law of throughput capacity in the uplink scenario of VANETs. Our results show that in both free-space propagation and non-freespace propagation environments, the achievable throughput capacity per vehicle can scale as $\Theta\left(\frac{1}{\log n}\right)$ where $n$ denotes the population of a homogenous set of vehicles in the network ${ }^{2}$.

Second, for a large number of vehicles in urban areas, the heavy data traffic makes it necessary to deploy sufficient RSUs

[^2]in the network to provide guaranteed throughput performance to users. However, overly deployed RSUs will incur high implementation and maintainable cost. Thereby RSU deployment should be carefully designed to effectively solve this tradeoff. By answering $Q 2$ we show that, to attain the asymptotic throughput capacity $\Theta\left(\frac{1}{\log n}\right)$, the number of effective RSUs should scale as $\Theta\left(\frac{n}{\log n}\right)$. This result can serve as the valuable benchmark for the real-world RSU deployment and service provisioning.

Finally, to address $Q 3$, we develop a novel packet forwarding scheme to approach the asymptotic throughput capacity in VANETs. Since the data traffic generated by vehicles can be highly unbalanced in real-world ${ }^{3}$, the RSUs cannot be evenly and fully used (i.e., some RSUs are overloaded whereas others are light-loaded without much traffic to deliver), resulting in poor network throughput performance. The proposed scheme makes full use of the mobility diversity of vehicles, to realize load-balanced utilization of RSUs and therefore enhance network throughput. In specific, the source vehicle selects the nearby vehicles whose mobility metrics (in moving direction and velocity) are significantly different from the source's as the relays for inter-vehicle transmissions. The more salient difference that the mobility metrics of two vehicles have, the higher probability that these two vehicles can exploit different RSUs will be. As a result, the data traffic can be balanced throughout the network, and more concurrent uploading opportunities from vehicles to different RSUs can be created, which leads to the improved throughput of uploading sessions in the system-wide.

The remainder of this paper is organized as follows: Section II surveys the related work. The system model and the notion of throughput capacity are introduced in Section III. Section IV analyzes the asymptotic throughput capacity of the defined network. A mobility diversity-based forwarding scheme is proposed in Section V. Section VI presents performance evaluations of the proposed forwarding scheme based on realistic traces. Finally, Section VII concludes the paper.

## II. Related Work

The theoretical asymptotic throughput capacity was first investigated in specific scenarios of wireless networks. In [8], Gupta and Kumar have proved that the nodal throughput capacity of static nodes with multihop relays diminishes to zero as $\Theta\left(\frac{1}{\sqrt{n}}\right)$ with $n$ denoting the node population. Grossglauser and Tse have extended the results in [9] showing that the constant throughput capacity is achievable when considering the extreme mobility. However, the constant per-node throughput capacity is achieved at the cost of larger delay. To address this issue in MANETs with practical models, Neely and Modiano have unveiled the tradeoff between throughput capacity and delay for a cell-partitioned ad hoc network in [15]. Under different scheduling policies, it is uncovered that although the

[^3]scheduling policies can exploit packet redundancy to reduce delay, redundancy packets degrade throughput capacity as well. In [16], with a more practical restricted random mobility model, Li et al. have achieved a smooth tradeoff between throughput capacity and delay by controlling node mobility. Furthermore, in [17], given the node and spatial heterogeneity, Garetto et al. have provided a general framework on the analysis of the throughput capacity scaling laws in MANETs. In [18], Garetto and Leonardi have further demonstrated that when the mobile nodes are heterogenous with a restricted random mobility model in the network, the per-node throughput capacity can scale with a constant, and so can the delay.

Although VANETs are considered as a subgroup of MANETs, VANETs exhibit distinct features in terms of the network architecture, user mobility patterns, vehicle density, etc. Taking these features into consideration, the aforementioned research results derived in the general MANETs cannot be applied directly to VANETs. As the applications in VANETs are heterogeneous and rely on various communication patterns, current studies on the throughput capacity of VANETs are mainly application (or scenario)-driven [19]. For instance, for a vehicular transmission link, in [20], Scheuermann et al. have shown that any dissemination mechanism should change asymptotically faster than the changing pattern of $\frac{1}{a^{2}}$, where $a$ is the distance between the source vehicle and the destination, to guarantee the capacity to be scalable in a general setting. To improve the throughput of the network, access points (e.g., road side WiFi [14], parking vehicles [21], [22]) have been considered to increase the connectivity, however, these access points are expensive and not efficient. Instead, RSUs are considered to be the efficient, timely, and cheap solutions to the throughput improvement [23]. Specifically, in [24], Abdrabou and Zhuang have derived the effective throughput capacity of the V2R communications and evaluated the end-to-end delay performance between a vehicle and the nearest RSU. In [13], Lochert et al. have proposed an optimal deployment of RSUs to minimize the required bandwidth with the consideration of travel time savings for vehicles. In [10], [11], Nik et al. have studied the distance-limited capacity with (or without) RSUs, based on vehicle moving along a road. In [12], considering a network with restricted vehicle mobility in a certain mobile area, Lu et al. have shown that it is possible to achieve constant throughput capacity and constant delay based on a two-hop forwarding scheme. Most of these works are based on some specific mobility limitations in space, however, a comprehensive analysis on the general vehicular mobility model with more space freedom is still unavailable. By considering a more general mobile scenario, this paper is devoted to characterizing the achievable throughput capacity, and proposing a packet forwarding scheme based on relay selection to make the per-vehicle throughput approach the throughput capacity. Our previous paper [25] has investigated this subject but only gives the throughput lower bound in free-space propagation. In this work, the extensions are as follows: i) the transmission interference is calculated more accurately by taking into account not only the concurrent in-
terfering V2R transmissions but also the concurrent interfering V2V transmissions; ii) both the free-space and non-free-space propagation cases are derived accurately and separately; iii) the achievability of the throughput lower bounds are proved for the tightness of the bounds; iv) the derived lower bounds are further proved to be the throughput capacity given that the RSUs are deployed according to the derived scaling law; and $v$ ) the performance of the proposed forwarding scheme is evaluated more thoroughly via simulations.

## III. System Model

In this paper, we focus on data uploading scenario, in which data are generated at some source vehicles and destined to RSUs. Each vehicle can become a source vehicle with a predefined probability, and the source vehicles always have data to upload. In such a scenario, many applications can be enabled by efficient inter-vehicle transmissions, e.g., data uploading, email transmission and traffic information reporting. A summary of the important mathematical notations used in the paper is given in Table I.

We consider a general urban area where streets/roads are randomly distributed throughout the city with random lengths and directions [26]. The considered area is normalized to a unit $(1 \times 1)$ square with the left lower vertex denoted as the origin $(0,0)$ and the right upper vertex as $(1,1)$. Let $\mathbb{V}$ denote the set of vehicles (nodes) moving in the area with the population $n$, and vehicles are homogenous with the same node mobility model and transmission model. A set of $Q(n)$ RSUs are deployed uniformly, satisfying $Q(n)<n$. Since, in practice, the RSU deployments tend to be strategically deployed and therefore follow certain specific patterns, the grid topology can be a good option to study the uniform one as depicted in Fig. 1. However, the derivation process of the scaling laws in the paper is not restricted to the deterministic RSU deployment but based on the scenario when RSUs are randomly deployed under the uniform distribution.

Node Mobility Model: To obtain the analytical results, the Voronoi model [27] is applied to characterize vehicle mobility. In this model, vehicles are allowed not only to move on the street but also to stop along the street or in a building (e.g., a parking lot), thus a uniform distribution is adopted to model the vehicle node distribution [27]. Accordingly, the density of vehicles is $\rho(n)=\Theta(n)$. In a more general scenario when vehicles are non-uniformly distributed under some conditions, the network can be considered as a collection of several subregions. And in each subregion, vehicles are uniformly distributed with a different density compared with other subregions. To make the analysis tractable, we do not consider this scenario in this work. Although the uniform distribution is not realistic in all scenarios, the capacity derived can provide upper bound worthy of reference in realistic scenarios and be approached by exploiting efficient data forwarding schemes by balancing the traffic over the entire network.

Transmission Model: A transmission (V2V or V2R) is successful only if the received signal-to-interference-and-noise

TABLE I: A summary of important mathematical notations.

| Symbol | Description |
| :---: | :---: |
| V | The set of vehicles in the network |
| $n$ | The number of vehicles in the network |
| $Q(n)$ | The number of RSUs in the network |
| $\rho(n)$ | Vehicle density |
| $N_{0}$ | The Gaussian noise power |
| $G_{i j}$ | The path loss between vehicles $i$ and $j$ |
| $d_{i j}$ | The distance between vehicles $i$ and $j$ |
| $\beta$ | SINR threshold |
| $\alpha$ | The path loss exponent |
| $\varsigma$ | The upper bound of interference for a successful transmission |
| $R(n)$ | The radius of an RSU's guaranteed zone |
| $\pi(n)$ | The scheduling and relaying policy for all vehicles |
| $M_{j}^{\pi(n)}(t)$ | The number of packets received by RSUs from vehicle $j$ at time slot $t$ |
| D | The average distance between two adjacent RSUs |
| $d$ | The average distance between two neighboring vehicles |
| $k$ | The average distance between two neighboring RSUs normalized by the average distance between two adjacent vehicles |
| $I$ | The total interference for an RSU-receiver |
| $I_{V 2 R}$ | The interference at an RSU-receiver by other concurrent V2R transmissions |
| $I_{V 2 V}$ | The interference at an RSU-receiver by other concurrent V2V transmissions |
| $S_{i}$ | Set of the $i^{\text {th }}$ tier of RSU-neighboring zone |
| $r$ | The radius of a vehicle's transmission range normalized by the average distance between two adjacent vehicles |
| $\lambda(n)$ | The average nodal throughput |
| $\Upsilon$ | The Euler-Mascheroni constant |
| $X_{i}$ | The set of guaranteed zones of concurrent V2V transmissions around one $V 2 R$ transmission at the $i^{\text {th }}$ tier |
| $X_{i}^{\text {max }}\left(X_{i}^{\text {min }}\right)$ | The largest (smallest) set of possible concurrent-V2V-transmission guaranteed zones at the $i^{\text {th }}$ tier |
| $l$ | The normalized distance between the receivers of two adjacent concurrent-V2V-transmission guaranteed zones |
| $B$ | The average distance between the centers of two neighboring vehicle guaranteed zones |
| $f(n)$ | The possible number of concurrent V2R transmissions in one RSU guaranteed area at one time slot |
| $w$ | The probability of a vehicle as a source node |
| $P$ | The probability that there is at least one vehicle with data to transmit in an RSU's guaranteed zone |
| $W(n)$ | The Lambert W Function |
| $h$ | The number of hops in an end-to-end path |
| CT | The number of concurrent transmissions |

ratio (SINR) is no less than a threshold $\beta$, i.e.,

$$
S I N R=\frac{G_{i j} P_{i}}{N_{0}+\sum_{u \neq i} G_{u j} P_{u}} \geq \beta
$$



Fig. 1: RSUs deployment when $R(n)=k d / 2$.
where $N_{0}$ is the Gaussian noise power, $G_{i j}=d_{i j}{ }^{-\alpha}$ is the path loss between nodes $i$ and $j$, with $d_{i j}$ denoting the distance between two nodes and $\alpha$ denoting the path loss exponent [28]. Here, we consider that each node transmits with the same power, i.e., $P_{i}=P_{j}, i \neq j$. As interference power is usually much larger than noise power in the network of moderate/high density [29], we neglect noise effect to simplify the analysis, i.e.,

$$
\begin{equation*}
S I R=\frac{G_{i j}}{\sum_{u \neq i} G_{u j}} \geq \beta \tag{1}
\end{equation*}
$$

The received interference is limited to a channel quality related threshold $\varsigma \triangleq \frac{G_{i j}}{\beta}$. In data uploading scenario, we associate each receiver $j$ (an RSU or a vehicle) with an interferencerelated non-overlapping guaranteed zone in which there is no other receivers to be scheduled simultaneously to guarantee a successful reception. Denote the achievable radius of one RSU's guaranteed zone as $R(n)$. Under the scheduling policy, for receiver $j$, it is always possible to find an $R(n)$ which is large enough to limit interference lower than a certain value $\varsigma$, i.e., $\sum_{u \neq i} G_{u j} \leq \varsigma$, by properly controlling the number of neighboring vehicles in transmission. Therefore, the constraint of receiver's guaranteed zone can be used as another representation of the SIR requirement in (1).

Forwarding Scheme Overview: Due to the transient and intermittent connectivity among vehicles and RSUs, the data uploading processing of each source vehicle is inevitably interrupted from time to time. Therefore, it is desirable to chop data into multiple non-overlapping small packets and detach the small packets separately to different RSUs through different relaying vehicles once the RSUs are within the vehicles' transmission range. When all the packets from a chopped data are delivered to RSUs, the data uploading is completed.

As reported in [30], a two-hop routing strategy is enough to guarantee efficient utilization of the system bandwidth, if the
network throughput is highly balanced throughout the contact graph formed by the concurrent transmission links among vehicles and RSUs. Thus, in our work, the uploading of each chopped packet relies on a two-hop store-carry-forwarding scheme where V2V relaying is allowed to help the source vehicles to forward data to one RSU. In other words, each packet delivery involves at most one intermediate relaying vehicle before reaching an RSU. Once a packet is sent to a relay or an RSU successfully, the source vehicle evicts the packet from its buffer to avoid packet redundancy. In our setting, the contact duration between each transmission pair (e.g., V2V or V2R) is considered long enough to accomplish one packet delivery, which can be achieved by appropriately setting the packet size.

Definition of Throughput Capacity: To define asymptotic throughput capacity in our framework, let $\pi(n)$ denote the scheduling and relaying policy for all vehicles in the network. Time is partitioned into equal intervals, each interval being referred to as a time slot. During $T$ time slots, $M_{j}^{\pi(n)}(t)$ is denoted as the number of packets received by RSUs from vehicle $j$ at time slot $t, t \in(0, T]$ and $j \in \mathbb{V}$. With the random trajectories of vehicles on the streets, a long-term throughput capacity $\lambda(n)$ (packet/s) under a policy $\pi(n)$ is defined as

$$
\begin{equation*}
\lambda(n)=\min _{j}\left\{\lim _{T \rightarrow \infty} \inf \frac{1}{T} \sum_{t=1}^{T} M_{j}^{\pi(n)}(t)\right\} \tag{2}
\end{equation*}
$$

Specifically, when the vehicles in the network are homogenous, all the vehicles are with the same long-term average throughput performance, and thus the long-term average throughput capacity of $\lambda(n)$ in equation (2) is equal to the average throughput capacity of each vehicle over a long time, i.e.,

$$
\begin{align*}
\lambda(n) & =\lim _{T \rightarrow \infty, n \rightarrow \infty} \inf \frac{1}{n T} \sum_{j=1}^{n} \sum_{t=1}^{T} M_{j}^{\pi(n)}(t)  \tag{3}\\
& =\lim _{T \rightarrow \infty} \inf \frac{1}{T} \sum_{t=1}^{T} M_{j}^{\pi(n)}(t), \forall j \in \mathbb{V}
\end{align*}
$$

## IV. Asymptotic Analysis of Throughput Capacity

In this section, we present an analytical model to evaluate the asymptotic throughput capacity of VANETs. First, by calculating the maximal possible interference that an RSU suffers, the bound of asymptotic throughput capacity is derived for both free-space and non-free-space propagation environments. Then, the achievability of the derived upper bound is proved. Finally, the derived asymptotic throughput capacity is compared with the existing scaling laws of throughput capacity to elaborate its features.

Based on the concept of guaranteed zone mentioned in the previous section, let $N$ represent the average number of vehicles served by an RSU, which can be expressed as

$$
\begin{equation*}
N=\rho(n) \cdot \pi \cdot R^{2}(n) \tag{4}
\end{equation*}
$$

where $\rho(n)$ is the density of vehicles as defined in Section III.

The average distance between two adjacent RSUs and the average distance between two neighboring vehicles are denoted by $D$ and $2 d$, respectively. Since $n$ vehicles are uniformly distributed in a unit area, we have $d=\Theta\left(\frac{1}{\sqrt{n}}\right)$. Thus, the average area that each vehicle occupies is approximately $\pi d^{2}$. With the considered RSU deployment (see Fig. 1), by connecting the points of neighboring RSUs, we have a square area of $2 D^{2}$ for a central $\operatorname{RSU}$ (e.g., $R S U_{5}$ ). The square is composed of two circles (i.e., one circle and four quadrants) and the area among those circles. Then, the average number of vehicles in a guaranteed zone can also be approximated by

$$
\begin{equation*}
N=\frac{2 D^{2}}{\pi d^{2}} \cdot \frac{1}{2} \cdot \frac{1}{c_{1}}=\frac{1}{c_{2}} \cdot \frac{D^{2}}{d^{2}}=\frac{k^{2}}{c_{2}}, k \in\left(0, \sqrt{c_{2} n}\right] \tag{5}
\end{equation*}
$$

where $c_{1}(>1)$ is a constant to compensate the area when padding the square area with guaranteed zones, $c_{2}$ is a constant to simplify the derivation, and $k=D / d$ is the average distance between two neighboring RSUs normalized by the average distance between two adjacent vehicles.

## A. Upper Bound of Interference

The throughput capacity of the network is represented by the total number of concurrent V2R transmissions in the network. One transmission suffers the interference caused by other concurrent V2R/V2V transmissions. Furthermore, according to [31], the receiving RSU in the center of the network (e.g., $R S U_{5}$ ) faces at most four times the amount of interference that an RSU at the corner (e.g., $R S U_{1}$ ) is exposed to. Thus, the interference of a receiving RSU at the network center has the same order as that of the RSU at the corner, and we first consider the interference that an RSU in the corner suffers (e.g., $R S U_{1}$ ). The interference (denoted as $I$ ) that $R S U_{1}$ suffers is given by

$$
\begin{equation*}
I=I_{V 2 R}+I_{V 2 V} \tag{6}
\end{equation*}
$$

where $I_{V 2 R}$ is the interference of $R S U_{1}$ by other concurrent V2R transmissions, and $I_{V 2 V}$ is the interference of $R S U_{1}$ generated by concurrent V 2 V transmissions.

1) Interference Suffered from V2R Transmissions: We first study $I_{V 2 R}$. For the sake of simplicity, we only consider the receiving RSU at the left lower corner $(0,0)$ (i.e., the position of $R S U_{1}$ in Fig. 1). For $R S U_{1}$, let the set $S_{1}=\left\{R S U_{2}, R S U_{4}, R S U_{5}\right\}$ be the first tier of its neighboring RSU guaranteed zones, and $S_{2}=\left\{R S U_{3}, R S U_{6}, R S U_{7}, R S U_{8}, R S U_{9}\right\}$ be the second. We use $S_{i}$, satisfying $\left|S_{i}\right|=2 i+1$, to represent the set of the $i^{t h}$ tier of its neighboring zone. For the defined RSU deployment, $i$ should be less than $\sqrt{Q(n)}$, thus being less than $\sqrt{n}$ as well. For a transmitting vehicle located in the RSU's coverage range of the $i^{t h}$ tier, the normalized distance between $R S U_{1}$ and one vehicle is at least $(k \cdot i-r)$, where $r\left(<\frac{k}{2}\right)$ is the normalized radius of a vehicle's transmission range $T_{r}$ by the average distance between two adjacent vehicles $d$. According to (2), $\lambda(n)$ is the inferior nodal throughput, i.e., $\lambda(n)$ should be obtained under interference upper bound. Therefore, accumulating the interference from every zone tier,
we have

$$
\begin{equation*}
I_{V 2 R} \leq \sum_{i=1}^{\sqrt{Q(n)}} \frac{2 i+1}{(k i-r)^{\alpha}} \tag{7}
\end{equation*}
$$

where $\alpha$ is path loss exponent. Given $Q(n)<n$, we can further derive $I_{V 2 R}$ in (7) as follows

$$
\begin{aligned}
& I_{V 2 R} \leq \sum_{i=1}^{\sqrt{Q(n)}} \frac{2 i+1}{(k i-r)^{\alpha}}<\sum_{i=1}^{\sqrt{n}} \frac{2 i+1}{(k i-r)^{\alpha}} \\
&=\frac{2}{k^{\alpha}} \sum_{i=1}^{\sqrt{n}} \frac{i}{\left(i-\frac{r}{k}\right)^{\alpha}}+\frac{1}{k^{\alpha}} \sum_{i=1}^{\sqrt{n}} \frac{1}{\left(i-\frac{r}{k}\right)^{\alpha}} \\
&=\frac{2}{k^{\alpha}} \sum_{i=1}^{\sqrt{n}} \frac{1}{\left(i-\frac{r}{k}\right)^{\alpha-1}}+\left(\frac{1}{k^{\alpha}}+\frac{r}{k} \cdot \frac{2}{k^{\alpha}}\right) \sum_{i=1}^{\sqrt{n}} \frac{1}{\left(i-\frac{r}{k}\right)^{\alpha}} \\
& \quad<\frac{2}{k^{\alpha}}\left[\frac{1}{\left(1-\frac{r}{k}\right)^{\alpha-1}}+\sum_{i=2}^{\sqrt{n}} \frac{1}{(i-1)^{\alpha-1}}\right]+\left(\frac{1}{k^{\alpha}}+\frac{r}{k} \cdot \frac{2}{k^{\alpha}}\right) . \\
& {\left[\frac{1}{\left(1-\frac{r}{k}\right)^{\alpha}}+\sum_{i=2}^{\sqrt{n}} \frac{1}{(i-1)^{\alpha}}\right] } \\
& \quad<c_{3}+\frac{c_{4}}{k^{\alpha}} \sum_{i=1}^{\sqrt{n}-1} \frac{1}{i^{\alpha-1}}
\end{aligned}
$$

where $c_{3}=\frac{2}{k^{\alpha}} \cdot \frac{1}{\left(1-\frac{r}{k}\right)^{\alpha-1}}+\left(\frac{1}{k^{\alpha}}+\frac{r}{k} \cdot \frac{2}{k^{\alpha}}\right) \cdot \frac{1}{\left(1-\frac{r}{k}\right)^{\alpha}}$ a $c_{4}=3+\frac{2 r}{k}$ are constant to simplify the derivation; $\sum_{i=1}^{\sqrt{n}-1} \frac{1}{i^{\alpha-1}}$ is an $(\alpha-1)$-series. If we rearrange equation (7) based on the consideration $Q(n)<n$, we can obtain the derivation in (8) directly. Note that the above results are achieved under the condition, $r<k$, which holds for large $n$. If $\alpha=2$, the ( $\alpha-1$ )-series follows the diverging harmonic series: $\sum_{i=1}^{\sqrt{n}-1} \frac{1}{i}=\ln (\sqrt{n}-1)+\frac{1}{2(\sqrt{n}-1)}+\Upsilon$, where $\Upsilon$ is the EulerMascheroni constant; if $\alpha>2$, the $(\alpha-1)$-series converge to the Riemann zeta function: $\xi(\alpha-1)$ [31]. Then, (8) can be further simplified as

$$
I_{V 2 R}<\left\{\begin{array}{l}
c_{3}+\frac{c_{4}}{k^{2}}\left[\ln (\sqrt{n}-1)+\frac{1}{2(\sqrt{n}-1)}+\Upsilon\right], \text { if } \alpha=2  \tag{9}\\
c_{3}+\frac{c_{4}}{k^{\alpha}} \cdot \xi(\alpha-1), \text { if } \alpha>2
\end{array}\right.
$$

Note that $\left(\frac{1}{2(\sqrt{n}-1)}+\Upsilon\right)$ has no impact on the scaling law. Then,

$$
I_{V 2 R}<\left\{\begin{array}{l}
c_{3}+\frac{c_{4}}{2 k^{2}}[\ln n+o(\ln n)], \text { if } \alpha=2  \tag{10}\\
c_{3}+\frac{c_{4}}{k^{\alpha}} \xi(\alpha-1), \text { if } \alpha>2
\end{array}\right.
$$

2) Interference Suffered from V2V Transmissions: On the other hand, the concurrent V2V transmissions also contribute to the interference that an RSU suffers. Similar to RSUs, we consider that each receiving vehicle also has a guaranteed zone which is disjoint with each other. Thus, the transmission from vehicle $i$ to vehicle $j$ is successful only when the following condition holds: $d_{u j} \geq(1+\Delta) r$, where $d_{u j}$ is the Euclidean distance between other transmitting vehicle $u$ and receiver $j$; $\Delta(>0)$ is a guard factor. Then the network can be divided into $n$ small disjoint guaranteed zones, and there is only one vehicle in each zone.

Let the set $X_{i}$ be composed of guaranteed zones for
concurrent V 2 V transmissions around one $V 2 R$ transmission at the $i^{\text {th }}$ tier, and $V 2 V_{j}(j=1,2, \ldots, n / 2)$ represents the guaranteed zone for the $j^{\text {th }}$ concurrent V2V transmission. For one guaranteed zone of the RSU, several concurrent- $V 2 V$-transmission guaranteed zones are around. Let $X_{i}^{\max }$ and $X_{i}^{\min }$ respectively denote the largest and the smallest sets of possible concurrent-V2V-transmission guaranteed zones at the $i^{t h}$ tier. Take the first tier as an example: considering an interested $V 2 R$ transmission (e.g., $V 2 R_{0}$ in Fig. 2), the set $X_{1}^{\max }$ is the largest set of neighboring concurrent-V2V-transmission guaranteed zones at the first tier. There are at most six concurrent V2V transmission guaranteed zones around one V 2 R transmission, i.e., $X_{1}^{\max }=\left\{V 2 V_{1}, V 2 V_{2}, V 2 V_{3}, V 2 V_{4}, V 2 V_{5}, V 2 V_{6}\right\}$. Similarly, the largest set of concurrent-V2V-transmission guaranteed zones at the second tier is $X_{2}^{\max }=$ $\left\{V 2 V_{7}, V 2 V_{8}, \ldots, V 2 V_{17}, V 2 V_{18}\right\}$. By extending this logic, $\left|X_{i}^{\text {max }}\right|=6 i$. The smallest set $X_{i}^{\text {min }}$ is shown as the shadow part when an RSU is set at the network corner in Fig. 2, satisfying $\left|X_{i}^{\text {min }}\right|<3 i$. The interference of one receiving RSU which is at the center, is with the same order as that of an RSU at the corner. Here we still use $R S U_{1}$ in Fig. 1 as the receiving RSU for interference analysis. Then, the normalized distances between the receiver of every concurrent V2V transmission in the $i^{t h}$ tier and $R S U_{1}$ are no less than $\frac{\sqrt{3}}{2} l i$ $(l \in(0, \sqrt{n}])$, where $l=\frac{B}{d}$ is the normalized distance between the receivers of two adjacent concurrent-V2V-transmission guaranteed zones, and, $B$ is the average distance between the centers of two neighboring vehicle guaranteed zones. Then, we have

$$
\begin{align*}
I_{V 2 V} \leq & \sum_{i=1}^{\sqrt{n}} \frac{3 i}{\left(\frac{\sqrt{3}}{2} l i-r\right)^{\alpha}} \leq \frac{3}{\left(\frac{\sqrt{3}}{2} l\right)^{\alpha}} \sum_{i=1}^{\sqrt{n}} \frac{i-\frac{2 r}{\sqrt{3}}+\frac{2 r}{\sqrt{3} l}}{\left(i-\frac{2 r}{\sqrt{3} l}\right)^{\alpha}} \\
= & \frac{3}{\left(\frac{\sqrt{3}}{2} l\right)^{\alpha}} \sum_{i=1}^{\sqrt{n}} \frac{1}{\left(i-\frac{2 r}{\sqrt{3} l}\right)^{\alpha-1}}+\frac{3}{\left(\frac{\sqrt{3}}{2} l\right)^{\alpha}} \cdot \frac{2 r}{\sqrt{3} l} \sum_{i=1}^{\sqrt{n}} \frac{1}{\left(i-\frac{2 r}{\sqrt{3} l}\right)^{\alpha}} \\
< & \frac{3}{\left(\frac{\sqrt{3}}{2} l\right)^{\alpha}}\left[\frac{1}{\left(1-\frac{2 r}{\sqrt{3} l}\right)^{\alpha-1}}+\sum_{i=2}^{\sqrt{n}} \frac{1}{(i-1)^{\alpha-1}}\right]+ \\
& \frac{3}{\left(\frac{\sqrt{3}}{2} l\right)^{\alpha}} \cdot \frac{2 r}{\sqrt{3} l}\left[\frac{1}{\left(1-\frac{2 r}{\sqrt{3} l}\right)^{\alpha}}+\sum_{i=2}^{\sqrt{n}} \frac{1}{(i-1)^{\alpha}}\right] \\
< & c_{5}+\frac{c_{6}}{l^{\alpha}} \sum_{i=1}^{\sqrt{n}-1} \frac{1}{i^{\alpha-1}} \tag{11}
\end{align*}
$$

where $c_{5}=\frac{3}{\left(\frac{\sqrt{3}}{2} l\right)^{\alpha}} \cdot \frac{1}{\left(1-\frac{2 r}{\sqrt{3} l}\right)^{\alpha-1}}+\frac{3}{\left(\frac{\sqrt{3}}{2} l\right)^{\alpha}} \cdot \frac{2 r}{\sqrt{3} l} \cdot \frac{1}{\left(1-\frac{2 r}{\sqrt{3}}\right)^{\alpha}}$ and $c_{6}=\frac{3}{\left(\frac{\sqrt{3}}{2}\right)^{\alpha}}+\frac{3}{\left(\frac{\sqrt{3}}{2}\right)^{\alpha}} \cdot \frac{2 r}{\sqrt{3} l}$ are constants to simplify the derivation. $\left(\frac{\sqrt{3}}{2} l i-r\right)$ is the distance between $R S U_{0}$ and one transmitter on the $i^{t h}$ tier, satisfying $\frac{\sqrt{3}}{2} l-r>0$. Similar to (8), (11) can be further simplified as

$$
I_{V 2 V}<\left\{\begin{array}{l}
c_{5}+\frac{c_{6}}{2^{2}}[\ln n+o(\ln n)], \text { if } \alpha=2  \tag{12}\\
c_{5}+\frac{c_{6}}{l^{\alpha}} \xi(\alpha-1), \text { if } \alpha>2
\end{array}\right.
$$

Finally, combining (6), (10) and (12), we have the following


Fig. 2: An interested V2R transmission $\left(V 2 R_{0}\right)$ is scheduled in the receiver's guaranteed zone, while other possible concurrent transmissions ( $V 2 V_{i}, i=1,2,3 \ldots, n / 2$ ) occurring every $l$ distance away.
interference upper bound,

$$
I<\left\{\begin{array}{l}
c_{3}+\frac{c_{4}}{2 k^{2}}[\ln n+o(\ln n)]+c_{5}+\frac{c_{6}}{2 l^{2}}[\ln n+o(\ln n)]  \tag{13}\\
\quad \text { if } \alpha=2 \\
c_{3}+\frac{c_{4}}{k^{\alpha}} \xi(\alpha-1)+c_{5}+\frac{c_{6}}{l^{\alpha}} \xi(\alpha-1),
\end{array}\right.
$$

## B. Bounds of Per-vehicle Throughput Capacity

Since a transmission is successful only when the SIR exceeds the threshold $\beta$, to guarantee a successful transmission, the interference at a receiver should be bounded (as aforementioned in (1)). In other words, both $\frac{\ln n}{k^{2}}\left(\frac{\xi(\alpha-1)}{k^{\alpha}}\right)$ and $\frac{\ln n}{l^{2}}\left(\frac{\xi(\alpha-1)}{l^{\alpha}}\right)$ should be finite, leading to

$$
\begin{align*}
& k=\left\{\begin{array}{l}
\Omega(\sqrt{\ln n}), \text { if } \alpha=2 \\
\Omega\left((\xi(\alpha-1))^{\frac{1}{\alpha}}\right), \text { if } \alpha>2,
\end{array}\right. \\
& l=\left\{\begin{array}{l}
\Omega(\sqrt{\ln n}), \text { if } \alpha=2 \\
\Omega\left((\xi(\alpha-1))^{\frac{1}{\alpha}}\right), \text { if } \alpha>2 .
\end{array}\right. \tag{14}
\end{align*}
$$

Further, from (4), (5) and (14), we have

$$
\begin{align*}
& N=\left\{\begin{array}{l}
\Omega(\ln n), \text { if } \alpha=2 \\
\Omega\left((\xi(\alpha-1))^{\frac{2}{\alpha}}\right), \text { if } \alpha>2
\end{array}\right. \\
& R(n)=\left\{\begin{array}{l}
\Omega\left(\sqrt{\frac{\ln n}{n}}\right), \text { if } \alpha=2 \\
\Omega\left(\frac{(\xi(\alpha-1))^{\frac{1}{\alpha}}}{\sqrt{n}}\right), \text { if } \alpha>2
\end{array}\right. \tag{15}
\end{align*}
$$

Then, we have the Lemma 1.
Lemma 1: Within the network model defined in Section III, we have
i). For a free-space propagation environment (i.e., $\alpha=2$ ), when the number of RSUs $Q(n)$ scales as $\Theta\left(\frac{n}{\log n}\right)$, the throughput capacity $\lambda(n)$ scales as $O\left(\frac{1}{\log n}\right)$;
ii). For a non-free-space propagation environment (i.e., $\alpha>2$ ), when the number of RSUs $Q(n)$ scales as
$\Theta\left(\frac{n}{(\xi(\alpha-1))^{\frac{2}{\alpha}}}\right)$, the throughput capacity $\lambda(n)$ scales as $O\left(\frac{(\xi(\alpha-1))^{\alpha}}{(\xi(\alpha-1))^{\frac{2}{\alpha}}}\right)$, where $\xi(\alpha-1)$ is the Riemann zeta function at point $(\alpha-1)$.
Proof: The throughput capacity, $\lambda(n)$, determined by the total number of concurrent V2R transmissions in any time slot, should satisfy the following inequality,

$$
\begin{equation*}
n \cdot \lambda(n)=\sum_{j=1}^{n} M_{j}^{\pi(n)}(t) \leq Q(n) \leq \frac{n}{N} \tag{16}
\end{equation*}
$$

where the last inequality holds due to the fact that the guaranteed zones of all RSUs may not fully cover the whole area. Then, combining (15) and (16),

$$
Q(n)=\left\{\begin{array}{l}
O\left(\frac{n}{\ln n}\right), \text { if } \alpha=2  \tag{17}\\
O\left(\frac{n}{(\xi(\alpha-1))^{\frac{2}{\alpha}}}\right), \text { if } \alpha>2
\end{array}\right.
$$

For a free-space propagation environment, when the number of RSUs scales as $Q(n)=\Theta\left(\frac{n}{\ln n}\right)$, the throughput capacity is thus $\lambda(n)=O\left(\frac{1}{\ln n}\right)$. Note that $\ln n$ and $\log n$ are in the same scaling. Similarly, for a non-free-space propagation environment, when the number of deployed RSUs $Q(n)$ scales as $\Theta\left(\frac{n}{(\xi(\alpha-1))^{\frac{2}{\alpha}}}\right)$, the throughput capacity $\lambda(n)$ scale as $O\left(\frac{1}{(\xi(\alpha-1))^{\frac{2}{\alpha}}}\right)$ accordingly, i.e.,

$$
\lambda(n)=\left\{\begin{array}{l}
O\left(\frac{1}{\ln n}\right), \text { if } \alpha=2 \\
O\left(\frac{1}{(\xi(\alpha-1))^{\frac{2}{\alpha}}}\right), \text { if } \alpha>2
\end{array}\right.
$$

Remark 1: Note that either $(\xi(\alpha-1))^{\frac{2}{\alpha}}$ or $\ln n$ is in a smaller order compared to $n$, and thus for both cases $Q(n)$ will also go to infinity as $n$ does. Therefore, we can use $n$ instead of $Q(n)$ in the inequality amplification of (8) to keep a tight bound.

Remark 2: $\lambda(n)$ is the largest lower bound of the throughput for every vehicle. Specifically, in the studied homogeneous case, $\lambda(n)$ equals to the average throughput capacity. On the other hand, for uploading applications, the number of concurrent transmissions is limited by the number of RSUs, and thus the average throughput should be upper bounded by $Q(n) / n$. If the constraint of the throughput upper bound, i.e., the number of RSUs, scales as $Q(n)=\Theta\left(\frac{n}{\ln n}\right)$, the corresponding throughput is achievable. By the Squeeze Theorem [32], we can conclude that the average throughput capacity can scale as $\Theta\left(\frac{1}{\ln n}\right)$.

## C. Achievability Analysis

In the following subsections, we strictly prove the achievability of $\lambda(n)=\Theta\left(\frac{1}{\ln n}\right)$ when $Q(n)=\Theta\left(\frac{n}{\ln n}\right)$, for different fading scenarios characterized by path loss exponent $\alpha$. To prove the achievability, we first conclude the corollary below from Lemma 1.

Corollary 1: Within the network as aforementioned, there is at most one transmission (both V2R and V2V transmissions) in the guaranteed zone of one RSU when the number of RSUs scales as $\Theta\left(\frac{n}{\log n}\right)$.

Proof: Within a normalized area, when the number of RSUs scales as $\Theta\left(\frac{n}{\log n}\right)$, it can be deduced from the previous discussion that, the distance between any two neighboring receiving RSUs (i.e., $k$ ) scales as $\Theta(\sqrt{\log n})$, and the distance between the receivers of two neighboring concurrent V 2 V transmissions should satisfy $l=\Theta(\sqrt{\log n})$. Therefore, there could be only one V2R or V2V transmission (i.e., only one transmitting vehicle) scheduled in an RSU guaranteed zone, due to the constraint of finite interference.

Let $f(n)$ denote the possible number of concurrent V2R transmissions in one RSU guaranteed area at one time slot. We have the following lemmas and theorems.

1) Achievability in Free-Space Propagation Environment (for $\alpha=2$ ):

Lemma 2: Consider that $n$ vehicles are uniformly distributed in a normalized area. If the number of RSUs scales as $\Theta\left(\frac{n}{\log n}\right), f(n)$ is no less than one with high probability when $n$ goes to infinity for every RSU, i.e.,

$$
\left.\lim _{n \rightarrow \infty} \operatorname{Pr}(f(n) \geq 1)\right)=1, \forall \mathrm{RSU}
$$

Proof: We first give the proof for one single RSU. With the two-hop forwarding scheme, the probability that one vehicle has data to transmit is $w$. Because of the nodal uniform distribution, the probability of one vehicle having data to deliver in an RSU guaranteed zone is $w \cdot \frac{\pi \cdot R^{2}(n)}{1^{2}}$. Then the probability (denoted by $P$ ) that at least one vehicle with data transmits in the RSU's guaranteed zone is

$$
\begin{equation*}
P=1-\left(1-w \cdot \frac{\pi \cdot R^{2}(n)}{1^{2}}\right)^{n} \tag{18}
\end{equation*}
$$

With $n$ goes to infinity,

$$
\begin{equation*}
\lim _{n \rightarrow \infty}\left[1-\left(1-w \cdot \frac{\pi \cdot R^{2}(n)}{1^{2}}\right)^{n}\right]=1-\lim _{n \rightarrow \infty}\left(1-w \cdot \frac{\pi \cdot R^{2}(n)}{1^{2}}\right)^{n} \tag{19}
\end{equation*}
$$

Based on (15), we have

$$
\begin{align*}
& \lim _{n \rightarrow \infty}\left(1-w \cdot \frac{\pi \cdot R^{2}(n)}{1^{2}}\right)^{n} \\
& =\lim _{n \rightarrow \infty}\left(\left(1-w \cdot \frac{\pi \cdot a_{1} \cdot \frac{\ln n}{n}}{1^{2}}\right)^{\left(-\frac{n \cdot 1^{2}}{w \pi a_{1} \ln n}\right)}\right)^{\left(-\frac{w \pi a_{1} \ln n}{1^{2}}\right)} \tag{20}
\end{align*}
$$

where $a_{1}>0$ is a positive constant, and $\lim _{n \rightarrow \infty}\left(1-w \cdot \frac{\pi \cdot a_{1} \cdot \frac{\ln n}{n}}{1^{2}}\right)^{\left(-\frac{n \cdot 1^{2}}{w \pi a_{1} \ln n}\right)}=e$. Therefore,

$$
\begin{equation*}
\lim _{n \rightarrow \infty} P=\lim _{n \rightarrow \infty}\left[1-\left(1-w \cdot \frac{\pi \cdot R^{2}(n)}{1^{2}}\right)^{n}\right]=1 \tag{21}
\end{equation*}
$$

Next we prove that every RSU satisfies equation (21). Let event $A_{m}$ denote the event that there is at least one vehicle with data in the guaranteed zone of $R S U_{m}$ where $m=1,2, \ldots, Q(n)$. By definition, $P$ is the probability that at least one vehicle with data is in an RSU guaranteed zone, and thus $P\left(A_{m}\right)=P=1-P\left(\overline{A_{m}}\right)$. Further, $P^{Q(n)}$ is the probability that at least one vehicle with data is in the RSU guaranteed zone for every RSU, and $Q(n)$ scales as $\Theta\left(\frac{n}{\ln n}\right)$. From the union bound, we have

$$
\begin{align*}
P^{Q(n)} & =P\left(A_{1} \cap A_{2} \cap \ldots \cap A_{Q(n)}\right) \\
& =1-P\left(\overline{A_{1}} \cup \overline{A_{2}} \cup \ldots \cup \frac{A_{Q(n)}}{}\right) . \tag{22}
\end{align*}
$$

As $P\left(\overline{A_{1}} \cup \overline{A_{2}} \cup \ldots \cup \overline{A_{Q(n)}}\right) \leq \sum_{m=1}^{Q(n)} P\left(\overline{A_{m}}\right)$, (22) can be simplified as

$$
\begin{equation*}
P^{Q(n)} \geq 1-\sum_{m=1}^{Q(n)} P\left(\overline{A_{m}}\right)=1-Q(n) \cdot(1-P) \tag{23}
\end{equation*}
$$

where, based on (18), $\lim _{n \rightarrow \infty} Q(n) \cdot(1-P)=\lim _{n \rightarrow \infty} a_{2} \cdot \frac{n}{\ln n}$. $\left(1-w \cdot \frac{\pi \cdot R^{2}(n)}{1^{2}}\right)^{n}=0$, with $a_{2}$ being a constant to simplify the equation.

Therefore,

$$
\begin{equation*}
\lim _{n \rightarrow \infty} P^{Q(n)}=1 \tag{24}
\end{equation*}
$$

i.e., $\left.\lim _{n \rightarrow \infty} \operatorname{Pr}(f(n) \geq 1)\right)=1, \forall \operatorname{RSU}$.

Thus, from Lemma 1, Corollary 1, and Lemma 2, we have Theorem 1.

Theorem 1: For free-space propagation environment with $n$ vehicles uniformly distributed in a normalized area, when the number of RSUs deployed in the network scales as $\Theta\left(\frac{n}{\log n}\right)$, $f(n)$ converges to one in probability, for every RSU at the same time slot, i.e.,

$$
\lim _{n \rightarrow \infty} \operatorname{Prob}(|f(n)-1|>\varepsilon)=0, \forall \varepsilon>0, \text { and } \forall \mathrm{RSU}
$$

Therefore, the achievable throughput capacity scales as $\Theta\left(\frac{1}{\log n}\right)$.

In general, the vehicular communication environment rarely follows free-space prorogation model. Therefore, the throughput capacity derived for this channel model should be considered as a performance upper bound. However, it is interesting to learn that, for a fading scenario with path loss exponent larger than 2 , the appropriate setting of RSU can help to achieve this throughput capacity bound as well.
2) Achievability in Non-Free-Space Propagation Environment (for $\alpha>2$ ): In the non-free-space propagation environment, based on Lemma 1 when the number of RSUs scales as $\Theta\left(\frac{n}{(\xi(\alpha-1))^{\frac{2}{\alpha}}}\right)$, the throughput capacity should be $\lambda(n)=O\left(\frac{1}{(\xi(\alpha-1))^{\frac{2}{\alpha}}}\right)$, which however, is not achievable. The reason is that since $(\xi(\alpha-1))^{\frac{2}{\alpha}}$ is smaller than $\ln n$, the number of RSUs for $\alpha>2$ is larger than that for $\alpha=2$ according to (17). Constrained by the guaranteed zones of vehicles and RSUs, it cannot be ensured that there are at least one vehicle with data to transmit in every RSU's guaranteed zone with the increased number of RSUs. Next, we show the necessary conditions of RSU deployment to make the throughput capacity achievable.

Theorem 2: For non-free-space propagation environment, given $n$ vehicles in a normalized unit square area, when the number of RSUs $Q(n)$ scales as $O\left(\frac{n}{(\xi(\alpha-1))^{\frac{2}{\alpha}}}\right)$ and $Q(n)=o\left(e^{\frac{n \cdot w \pi}{Q(n)}}\right)$ holds, the throughput capacity $\Theta\left(\frac{Q(n)}{n}\right)$ can be achievable. More specifically, when the RSUs are deployed with $Q(n)=\Theta\left(\frac{n}{\log n}\right), f(n)$ is no less than one with high
probability for every RSU, i.e.,

$$
\lim _{n \rightarrow \infty} \operatorname{Prob}(f(n) \geq 1)=1, \forall \mathrm{RSU}
$$

Therefore, the achievable throughput capacity scales as $\Theta\left(\frac{1}{\log n}\right)$.

Proof: Let $P$ be the probability that at least one vehicle with data is in an RSU guaranteed zone. $P^{Q(n)}$ is the probability that at least one vehicle with data is in the RSU's guaranteed zone for every RSU. To maximize network throughput, the RSUs should be deployed to fully cover the whole network, hence $Q(n)=\Theta\left(\frac{n}{N}\right)$. Based on Lemma 1, $Q(n)=O\left(\frac{n}{(\xi(\alpha-1))^{\frac{2}{\alpha}}}\right)$, and combining (4), $Q(n)=\Theta\left(\frac{1}{R^{2}(n)}\right)$, we have

$$
\begin{align*}
& P=1-\left(1-w \cdot \frac{\pi}{Q(n)}\right)^{n} \\
& P^{Q(n)} \geq 1-Q(n) \cdot\left(1-w \cdot \frac{\pi}{Q(n)}\right)^{n} \tag{25}
\end{align*}
$$

where $\lim _{n \rightarrow \infty} Q(n) \cdot\left(1-w \cdot \frac{\pi}{Q(n)}\right)^{n}=\lim _{n \rightarrow \infty} Q(n)$. $\left(1-w \cdot \frac{\pi}{Q(n)}\right)^{\left(-\frac{Q(n)}{w \pi}\right) \cdot\left(-\frac{n w \pi}{Q(n)}\right)}$. Therefore, when the condition $Q(n)=o\left(e^{\frac{n \cdot w \pi}{Q(n)}}\right)$ holds, we can attain $\lim _{n \rightarrow \infty} P^{Q(n)}=$ 1. In other words, when the number of $\stackrel{n}{\mathrm{RSSUS}} \underset{n \cdot w \pi}{\infty}$ scales as $O\left(\frac{n}{(\xi(\alpha-1))^{\frac{2}{\alpha}}}\right)$ and the condition $Q(n)=o\left(e^{\frac{n}{Q(n)}}\right)$ holds, the throughput $\Theta\left(\frac{Q(n)}{n}\right)$ can be achievable.

As for $Q(n)$, the solution of equation $Q(n)=e^{\frac{n}{Q(n)}}$ is $Q(n)=\frac{n}{W(n)}$, where $W(n)$ is the Lambert $W$ Function [33]. More specifically $W(n)=\ln n-\ln W(n)$, where $\ln W(n)$ has a much less order compared to $\ln n$. If $Q(n)=$ $\frac{n}{\ln n}$, the above condition $Q(n)=o\left(e^{\frac{n \cdot w \pi}{Q(n)}}\right)$ can be satisfied. Therefore, for $\alpha>2$, if the number of RSUs $Q(n)$ scales as $\Theta\left(\frac{n}{\ln n}\right)$, the achievable throughput capacity can be $\Theta\left(\frac{1}{\ln n}\right)$.

This result is the same as the one for free-space environment . It comes from the constraint that the scaling of the distance between two adjacent RSUs must be no smaller than the order of $\Theta\left(\sqrt{\frac{\ln n}{n}}\right)$ to guarantee at least one vehicle with data in an RSU's guaranteed zone.

Remark 3: When RSUs are uniformly distributed with the same average distance between neighboring RSUs as that under grid pattern, the scaling law of uniform RSU deployment is the same as that under grid pattern RSU deployment. In addition, with a large number of RSUs, the randomness of the uniform deployment of RSUs has little impact on general trend of network performances. Thus, the proposed asymptotic throughput capacity throughout the paper holds for both uniform and grid pattern RSU deployments. And, the average throughput performances under uniform RSU deployment are the same as those under grid pattern RSU deployment with high probability.

## D. Discussion

Based on Theorem 1 and Theorem 2, we conclude that the achievable throughput capacity can scale as $\lambda(n)=\Theta\left(\frac{1}{\log n}\right)$ for both environments. The throughput capacity per node $\lambda(n)$
can be simply calculated as

$$
\begin{equation*}
n \times \lambda(n) \times h \leq C T \tag{26}
\end{equation*}
$$

where $h$ is the number of hops in an end-to-end path, $C T$ is the number of possible Concurrent Transmissions.

The throughput capacity $\lambda(n)=\Theta\left(\frac{1}{\sqrt{n}}\right)$ shown in [8] is based on the multihop forwarding scheme when all nodes are fixed in the network. Specifically, the number of hops scales as $\Theta(\sqrt{n})$, and the throughput capacity scales as $\lambda(n)=\Theta\left(\frac{1}{\sqrt{n}}\right)$. However, compared to the results shown in [8], the nodal mobility and the store-carry-forwarding scheme are adopted in our considered scenario, and therefore the derived throughput capacity is much higher than $\Theta\left(\frac{1}{\sqrt{n}}\right)$. On the other hand, the obtained throughput capacity $\lambda(n)=\Theta\left(\frac{1}{\log n}\right)$ is lower than the throughput capacity scaling law $\Theta(1)$ in [9]. That is because in [9] nodes are extremely mobile in the network, and one node is either a source node or a destination. At any time slot, $\Theta\left(\frac{n}{2}\right)$ concurrent transmissions can be guaranteed, and thus the number of the concurrent transmissions scales as $\Theta(n)$. Therefore, the throughput capacity can be a constant, $\Theta(1)$. Nevertheless, the asymptotic throughput capacity in this work is limited by the RSUs' deployment and vehicle mobility pattern, which result in the reduction of network throughput.

The most distinct feature of this work from the aforementioned ones is that we introduce RSUs as the destinations for all the end-to-end transmissions in the uploading scenarios. Thus, the throughput capacity bottleneck is the number of RSUs when the number of vehicles having data to transmit is huge. We show that when the number of RSUs $Q(n)$ scales as $O\left(\frac{n}{\log n}\right)$, the throughput capacity scales as $\Theta\left(\frac{Q(n)}{n}\right)$, i.e.,

$$
\begin{equation*}
\lambda(n)=\Theta\left(\frac{Q(n)}{n}\right), \text { if } Q(n)=O\left(\frac{n}{\log n}\right) \tag{27}
\end{equation*}
$$

The conclusion can serve as the valuable benchmark for real RSU deployment. With $n$ vehicles in the network, $\Theta\left(\frac{n}{\log n}\right)$ RSUs deployed in the network are enough to achieve the throughput capacity $\Theta\left(\frac{1}{\log n}\right)$, and the optimal available radius of an RSU guaranteed zone should scale as $\Theta\left(\sqrt{\frac{\log n}{n}}\right)$.

Note that the asymptotic throughput capacity is a theoretical throughput upper bound that can only be achieved when the vehicles are uniformly distributed through the network. In realworld VANET implementation, due to the street layout and the drivers' social characteristics, uniform distribution is hard to be achieved. Therefore, it is critical to have efficient and effective data forwarding schemes to balance the traffic to the whole network.

## V. Mobility Diversity-based Packet Forwarding Scheme

By exploiting mobility differentiation, more concurrent transmissions can be enabled to improve throughput performance approaching the asymptotic throughput capacity given in the previous section. To achieve this goal, a novel mobility diversity-based data forwarding scheme is proposed as follows.


Fig. 3: The moving path of vehicle $j$.

## A. Mobility Characteristics

As a vehicle may change its moving direction at an intersection following the street pattern, there are at most four scenarios (i.e., straight, right, left, and U-turn) for vehicles' turning options at each intersection in this paper. Consider that the starting point $S$, the destination $D$ and the path for each vehicle are known at the beginning. For example, as depicted in Fig. 3, a vehicle $j(j \in \mathbb{V})$ moves away from its starting point $S$ along its path with an initial velocity $v_{0 j}\left(\in\left[0, v_{\max }\right]\right)$ and moving direction $\varphi_{0 j}=\vartheta_{0 j}$ towards its destination $D$, where $\vartheta_{0 j} \in(-\pi, \pi]$ is the angle between the initial moving direction and the horizontal line. Let $\mathbb{M}_{j}$ denote the set of intersections that vehicle $j$ is passing through, where $m(\in \mathbb{M})$ represents the $m^{t h}$ intersection ( $m=1,2,3, \ldots$ ). For vehicle $j$ at the $m^{\text {th }}$ crossroad, if the deviation of moving direction from its current moving direction $\left(\varphi_{(m-1) j}\right)$ is represented by $\vartheta_{m j}(\in(-\pi, \pi])$ (see Fig. 3), the moving direction after the $m^{t h}$ crossroad $\left(\varphi_{m j}\right)$ is

$$
\begin{equation*}
\varphi_{m j}=\varphi_{0 j}+\sum_{i=1, i \in \mathbb{M}_{j}}^{m} \vartheta_{i j} \tag{28}
\end{equation*}
$$

where $\vartheta_{i j}$ has a positive (negative) value if the vehicle turns counterclockwise (clockwise).

The traveling time interval of one vehicle between two crossroads is defined as a time step, which depends on its current velocity and the length of the street. We use the current moving direction and velocity to describe the mobility characteristic of a vehicle in a time step. Let $\mathbb{B}$ denote the set of mobility characteristics of all vehicles, where each element $\mathbf{b}_{m j}(m \in \mathbb{M}, j \in \mathbb{V})$ represents the mobility characteristic of vehicle $j$ in its $m^{t h}$ time step along the path, including the vehicle's moving direction $\varphi_{m j}$ and current velocity $v_{m j}$, i.e., $\mathbf{b}_{m j}=\left(\varphi_{m j}, v_{m j}\right)$. The mobility characteristic remains the same for each vehicle within a time step.


Fig. 4: To measure the relative distance between $S$ and $R$.

## B. Mobility Differentiation Prediction

When a source vehicle $j$ arrives at an intersection (say at time $t_{0}$ ), it sends a request message to its neighboring vehicles. Upon receiving the request message, the neighboring vehicles will perform the algorithm for predicting their future positions at time $t_{0}+T_{p}$, and send the predicted positions back to the source vehicle $j$. Here $T_{p}$, called the prediction time in this work, is the time duration, specified by the source vehicle $j$. Based on the collected beforehand positions from the neighboring vehicles, vehicle $j$ will then calculate the relative distances between itself and the neighboring vehicles at time $t_{0}+T_{p}$, and make the decision on which neighbors are to be selected as relay nodes. Note that the communication time and calculation time are negligible compared to $T_{p}$. Both the concrete position predicting algorithm and the relay selection approach are described as follows.

As the city map can easily be acquired by GPS devices, the moving direction deviation at any intersection and the speed limits along the pre-set path can be obtained for each vehicle. If we use the speed limit of one street to estimate the average velocity of vehicles moving in the street, combined with equation (28), the vehicles' mobility characteristics along the streets can be predicted. As a result, the future position of one vehicle at time $t_{0}+T_{p}$ can be calculated. Taking Fig. 4 as an example, at time $t_{0}$, the source vehicle $S$ is in the $(m-1)^{t h}$ time step in position $\left(x_{s 1}, y_{s 1}\right)$, and it can communicate with one neighbor $R$ which is in its $(n-1)^{t h}$ time step in position $\left(x_{r 1}, y_{r 1}\right)$. The source vehicle $S$ moves with the mobility characteristic $\mathbf{b}_{(m-1) s}=\left(\varphi_{(m-1) s}, v_{(m-1) s}\right)$, while the neighboring vehicle $R$ moves with $\mathbf{b}_{(n-1) r}=\left(\varphi_{(n-1) r}, v_{(n-1) r}\right)$. After time $t_{1}, S$ reaches the $m^{t h}$ intersection in position $\left(x_{s 2}, y_{s 2}\right)=\left(x_{s 1}+v_{(m-1) s} \cdot t_{1} \cdot \cos \varphi_{(m-1) s}, \quad y_{s 1}+\right.$ $\left.v_{(m-1) s} \cdot t_{1} \cdot \sin \varphi_{(m-1) s}\right)$, while $R$ reaches its $n^{t h}$ intersection after time $t_{2}$ in position $\left(x_{r 2}, y_{r 2}\right)=\left(x_{r 1}+v_{(n-1) r} \cdot t_{2}\right.$. $\left.\cos \varphi_{(n-1) r}, y_{r 1}+v_{(n-1) r} \cdot t_{2} \cdot \sin \varphi_{(n-1) r}\right)$. Note that $t_{1}$ and $t_{2}$ can be calculated based on the respective current velocity and the distance from the vehicle to the next intersection. At time $t_{0}+T_{p}$, the positions of the vehicles (i.e., $x_{s 3}, y_{s 3}$, $x_{r 3}$, and $y_{r 3}$, respectively) can be obtained similarly, based on
$\left(x_{s 2}, y_{s 2}\right)$ and $\left(x_{r 2}, y_{r 2}\right)$, and therefore the relative distance between the vehicles can be attained. The relative distance $d_{s r}$ at time $t_{0}+T_{p}$ between this pair of vehicles is given as $d_{s r}=\sqrt{\left(x_{s 3}-x_{r 3}\right)^{2}+\left(y_{s 3}-y_{r 3}\right)^{2}}$.

## C. Mobility Diversity-based Forwarding Scheme

With the predicted relative distance $d_{s r}$ at time $t_{0}+T_{p}$, we can show the spatial differentiation among vehicles. When the source vehicle receives the neighbor vehicles' predicted locations, it transmits data only to the neighbors whose predicted relative distances are far enough from its own, i.e., exceeding one threshold (e.g., $2 R(n)$ ), to make full use of mobility differentiation. In other words, the source vehicle selects only appropriate relays whose mobility characteristics differ enough from its own to yield a diversity gain. Because this one-relay diversity is based on mobility differentiation, we call this diversity mobility diversity. The sketch of our proposed algorithm is summarized in Algorithm 1. The intrinsic philosophy behind this forwarding scheme is that by exploiting mobility diversity (e.g., line 27 in Algorithm 1), with the virtue of coordination, multiple vehicles can be used to reallocate the unbalanced data traffic and further allowed to concurrently transmit data packets generated by the same source vehicle to different RSUs, i.e., from line 25 to line 32 of Algorithm 1. In this way, RSUs' loads can be balanced to improve per-vehicle throughput performance.

In addition, two tracked parameters of mobility characteristics have different impacts on mobility differentiation in different scenarios. For instance, in urban areas, the velocities of vehicles are almost the same due to the relatively heavy vehicle-traffic density with little deviation from each other, while the moving directions vary greatly. Thus, the mobility differentiation can be reflected mainly on the nodal variation of moving directions. On the other hand, in the highway scenario, the moving direction deviation of a vehicle is limited by the highway pattern, while the velocity of the vehicle changes frequently. Therefore, the mobility differentiation in a highway scenario is mainly caused by differences in velocity.

## VI. Performance Evaluation

In this section, we show the case studies based on realistic traces collected from Shanghai taxis, to illustrate the derived scaling law and the performances of the proposed forwarding scheme with a discrete-event simulator developed in C++ language.

## A. Simulation Setting

The simulator implements the realistic vehicular traces which comprise the GPS locations of over 4000 Shanghai taxis within the 24 -hour period on Feb. 20th, 2010 [25]. In each simulation, we choose a predefined number of vehicle trace files with their trajectories pertained in the downtown area of Shanghai (with $5500 * 3000 \mathrm{~m}^{2}$ ), China. In our simulation, the simulated throughput is the average nodal throughput over 30 runs. For each run, the network throughput is equal to the cumulated number of packets received by RSUs over the

```
procedure Forwarding Scheme(Algorithm 1)
        /* Initialization */
        A set of vehicles \(\mathbb{V} \neq \emptyset\);
        A set of source vehicles \(I_{s}(\in \mathbb{V}) \neq \emptyset\);
        A set of neighboring vehicles \(I_{n}=\emptyset\);
        A candidate set of relays \(I_{c}=\emptyset\);
        /* Packet delivering */
        for each time slot \(t\) do
            for \(v \in\left\{\mathbb{V}-I_{s}\right\}\) do
            if vehicle \(v\) is with packets to transmit \& within the
range of an RSU then
                Upload one packet to the RSU directly when there
                        are no concurrent transmission requests from
                        other vehicles within the same RSU;
            end if
        end for
        for \(i \in I_{s}\) do
            if \(i\) is within the range of an RSU then
                Upload one packet to the RSU directly when there
                    are no concurrent transmission requests from
                    other vehicles within the same RSU;
            else
                        Build its own neighboring vehicle set \(I_{n}\) at time
\(t_{0} ; \quad\) while \(I_{n} \neq \emptyset\) do
                        if \(j\left(\in I_{n}\right)\) is not carrying any packet then
                                update the set \(I_{c} \leftarrow I_{c} \cup\{j\} ;\)
    end if
    \(I_{n} \leftarrow I_{n} \backslash\{j\} ;\)
                    end while
                    while \(I_{c} \neq \emptyset\) do
                            Choose any \(j \in I_{c}\);
                            Predict the mobility differentiation \(d_{i j}\) be-
                            tween vehicle \(i\) and vehicle \(j\) at time \(t_{0}+T_{P}\);
                            if \(d_{i j}>2 R(n)\) then
                                    \(j\) will receive the packet from \(i\);
    end if
    \(I_{c} \leftarrow I_{c} \backslash\{j\} ;\)
                    end while
            end if
        end for
    end for
end procedure
```

simulation period of 10800s. In addition, the mean throughput with $95 \%$ confidence intervals are considered in Figs. 6-10. RSUs are deployed uniformly. The transmission range of each vehicle is set to be 350 meters.

It should be notice that although in practice the taxi mobility is not adequate to represent the generic mobility of vehicles, studying the performances of vehicular communications in such scenarios is of interest and meaningful, because the penetration of vehicular communication would be a slow process, and at the initial stage of vehicular communications, it is very likely that a specific group of public vehicles would first be equipped with communication facility. For example, in both Shanghai and Beijing, a large number of taxis and buses have already been equipped with on-board GPS and light-weight communication devices [34]. Therefore, investigating the performance of vehicular communications in such scenarios is of interest and meaningful.

## B. Performance Evaluation

First, Fig. 5 presents that the simulated average nodal throughput performs the same trend with the derived analytical results. Here, vehicles are with the prediction time $T_{p}=300 \mathrm{~s}$. According to the theoretical results, if RSUs are deployed with the scaling as shown in Theorem 2, the theoretical average nodal throughput results follow a logarithmic fashion. As shown in Fig. 5, the simulated throughput performances present similar moderate/smooth decreasing trends with the theoretical scaling law, which supports the derived analytical results. Moreover, we can see that the proposed asymptotic throughput capacity holds under both the uniform and gridpatten deployments of RSUs, and the throughput performances under both deployment types are close to each other. The reasons are twofold. First, the grid pattern RSU deployment can illustrate the same scale of the distance between two adjacent RSUs under uniform distribution. Second, with a large number of RSUs, the randomness of the uniform deployment of RSUs has little impact on general trend of network performance. Thus, when RSUs are uniformly distributed with the same average distance between neighboring RSUs as that under grid pattern, the average throughput performances under uniform RSU deployment are the same as those under grid pattern RSU deployment with high probability. Besides, in practice, the RSU deployment tends to be strategically deployed by government or telecommunication companies and thus follows certain specific pattern. Therefore, considering both theoretical and simulation results, the grid topology can be a good option to study the performances of the scenario with uniform RSU deployment.

Then, we investigate the impacts of vehicle mobility on the average nodal throughput of the proposed scheme based on the collected realistic vehicular traces. The results are shown in Fig. 6 where the probability of each vehicle being a source (i.e., $w$ ) is set to be 0.1 , and the prediction time $T_{p}$ is set to be 300 s. In the simulation, we consider two different kinds of mobility patterns. In the first pattern, all vehicles in the network keep moving all the time without stopping throughout the simulation. For the second one, vehicles may stay still on road sides for a certain period of time and start moving again, e.g., when a taxi is waiting for customers outside hotels or shopping malls. It is conceivable that the mobility diversity among the vehicles of the first pattern is more striking than that of the second one. From Fig. 6, it can be seen that the average nodal throughput of the first mobility pattern is higher than that of the second one. This indicates that the average throughput increases with larger mobility diversity. The reason is that with larger mobility diversity, the source vehicle and the relay vehicles are more likely to have larger mobility differentiation after the same prediction time $T_{p}$. Thus, the relay vehicles are more likely to visit and deliver the packets to more different RSUs, creating more concurrent transmission opportunities. Besides, throughput comparison between the proposed scheme and the legacy two-hop forwarding scheme [9] is also given in Fig. 6. It can be observed from the figure


Fig. 5: Throughput performance with large number of vehicles.


Fig. 6: Throughput and RPR performance comparison when $w=0.1$.
that the proposed mobility-diversity-based packet forwarding scheme outperforms the legacy one by average $40 \%$ in terms of throughput, with different numbers of vehicles for both mobility patterns, due to more concurrent transmission opportunities introduced by the balanced traffic.

In Fig. 6, we also give a new performance metric Relayed Packet Ratio (RPR) to represent the weight of relays in contributing to the network throughput, which is calculated by the total number of uploaded packets from relay vehicles over the total number of uploaded packets during the whole simulation time. As shown in the figure, given $w=0.1$, with the increased number of vehicles, there are more chances for a source vehicle to find good relays for uploading, thus leading to a higher RPR. Since the candidates of good relays for source vehicles under mobility pattern I are more than those under mobility pattern II, the RPR can be higher in the former case.

Second, we compare the average nodal throughput of the proposed scheme with different values of prediction time $T_{p}$


Fig. 7: Throughput performance under mobility pattern II with different $T_{p}$ values ( $w=0.1$ ).


Fig. 8: Throughput performance under mobility pattern I with different values of $T_{p}(w=0.1)$.
under $w=0.1$, as shown in Fig. 7. Here, the mobility pattern II is used. It can be seen from Fig. 7 that the average nodal throughput achieved when $T_{p}$ is 300 s is higher than that when $T_{p}$ is 30 s . The reason is that with a smaller prediction interval (e.g., $T_{p}=30 \mathrm{~s}$ ), the differentiation of mobility characteristics among vehicles may not be large enough to fully exploit the mobility diversity, thus making it difficult for the source vehicle to accurately choose proper relays based on the predicted mobility differentiation. However, a larger $T_{p}$ does not always bring better performance. As shown in the figure, the performance is worse under a very large prediction time 2000s than that under 300s. This is because that if $T_{p}$ is too large, the chances to transmit before the end of predicted time can be neglected, and thus the selected relay may postpone the data delivery compared with a relay selected according to a shorter prediction time. The extended transmission delay reduces the network throughput accordingly.

Fig. 8 shows the throughput performance of the proposed scheme under mobility pattern I, in which all vehicles in the network keep moving all time without stopping throughout the simulation. It shows that the throughput performance under mobility pattern I is slightly higher than that under mobility pattern II on average. The reason is that in the mobility pattern I case, source vehicles have higher chances to find good relays with higher moving differentiations in order to make the generated data traffic as distributed throughout the network as possible, thus increasing the possible concurrent transmissions to RSUs. This also embodies the philosophy of our proposed forwarding scheme which utilizes the mobility differentiations of vehicles for choosing good relays to create more chances to meet RSUs for uploading. And this can also be considered to support that the average throughput increases with larger mobility diversity.

Furthermore, to evaluate the impact of another important parameter $w$ (i.e., the probability of a vehicle being a source) on the throughput performance, Figs. 9-10 are presented accordingly. Different from Fig. 6 where the throughput under both schemes increases with more vehicles when $w=0.1$, Fig. 9 shows that when $w=0.8$, the throughput under both schemes reduces as the number of vehicles increases. The reason is that with a small $w$, RSUs are far from saturated, and thus the increase of vehicles will bring more relaying opportunities for the sparsely distributed source vehicles and then create more concurrent V2R transmissions, resulting in a larger uploading throughput. On the other hand, for a large value of $w$, the data traffic volume generated in the network increases dramatically as the number of vehicles increases. In such a case, more and more RSUs become saturated and the network uploading throughput will increase more and more slowly. In consequence, the average nodal throughput will decrease with the the increased number of vehicles since it equals the ratio of the network throughput to the number of vehicles. Therefore, a large value of $w$ may have no positive impact on the average nodal throughput.

To illustrate more comprehensively how throughput changes with $w$, Fig. 10 is presented accordingly, given the number of vehicles $n=300$ and $n=900$. The numbers of RSUs for different $n$ are chosen respectively in order to facilitate the comparison. Given $n=300$, it can be seen that the average nodal throughput increases with $w$ until $w$ reaches 0.5 , and the throughput is almost unchanged when $w>0.5$. The reason is that when $w=0.1$ at $n=300$, there are not sufficiently large number of vehicles uploading concurrently to all the deployed RSUs. As $w$ increases, the number of source vehicles increases and there will be more concurrent V2R transmissions. As a result, the total uploading throughput increases, so does the average nodal throughput since the total number of vehicles is fixed. After $w$ reaches 0.5 , all the RSUs are saturated due to sufficiently large and balanced data traffic. Thus the total uploading throughput will be bounded by the number of RSUs and the average nodal throughput is then independent of $w$. When there are in total 900 vehicles, the maximum throughput will be quickly reached since there are more source vehicles


Fig. 9: Throughput performance comparison with different forwarding schemes $(w=0.8)$.


Fig. 10: Throughput and RPR performance comparison with different values of $w$.
compared with the former case (i.e., $n=300$ ), leading to more chances for both relaying and uploading. Therefore, as $w$ increases, the maximum throughput can be converged more quickly with a larger total number of vehicles.

Moreover, we simulate how the RPR changes with $w$, as shown in Fig. 10. It can be seen that for both cases, the RPR first increases and then decreases as $w$ increases. The reason is as follows: when $w$ is small, the neighboring relays are sufficient for the source vehicles. As $w$ increases, the number of packets transmitted to the relays increases faster due to the vehicle mobility diversity gain. As a result, the RPR increases first. When $w$ becomes large (e.g., $w>0.4$ ), on one hand, the percentage of the packets uploaded to the RSUs directly from source vehicles becomes higher, leading to a lower RPR. On the other hand, the possible relays, which have no packets for uploading, become insufficient. Since one relay can only carry one packet for one source vehicle, higher $w$ will increase
the conflict probability that the relay chosen by one source has already been chosen by another source, which will also decrease the RPR. Besides, the RPR of $n=300$ is first smaller and then larger than that of $n=900$. This is because when $w$ is small, relays are sufficient. Higher $n$ means there are more good relays to choose, thus having higher RPR. As $w$ becomes large and relays are not sufficient, higher $n$ can cause more conflicts in relay selection, thus leading to lower RPR.

## VII. Conclusions

In this paper, we have analyzed the asymptotic throughput capacity of VANETs for data uploading in urban areas. In both free-space propagation and non-free-space propagation environments, the achievable throughput capacity of VANETs scales as $\Theta\left(\frac{1}{\log n}\right)$ with the population of vehicles $n$, when the number of RSUs scales as $\Theta\left(\frac{n}{\log n}\right)$. This asymptotic throughput capacity can serve as a benchmark for real-world RSU deployment. In addition, a novel two-hop packet forwarding scheme has been proposed considering mobility diversity to approach the analytical throughput capacity in practice. For future work, we intend to investigate the concrete RSU deployment strategies in the light of our theoretical results on the RSU's scaling law, and optimize throughput performance by jointly considering the RSU deployment and intelligent forwarding schemes in not only urban areas but also freeways.
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[^1]:    ${ }^{1}$ In VANETs, many basic applications are supported in uplink scenario, such as data uploading, email transmission, road traffic reporting, and environment monitoring.

[^2]:    ${ }^{2}$ We consider two functions $f(x) \geq 0$ and $g(x) \geq 0$. The relationship between $f(x)$ and $g(x)$ is defined as $f(x)=O(g(x))$ or $g(x)=\Omega(f(x))$ if $\lim _{x \rightarrow \infty} \sup f(x) / g(x)=c<\infty . f(x)=\Theta(g(x))$ means $f(x)=$ $O(g(x))$ and $g(x)=O(f(x))$.

[^3]:    ${ }^{3}$ For example, the shuttle bus with many passengers on board may generate much more data traffic than sedans; the data generated in the parking lot of a shopping mall are more intensive than that generated in nearby residential areas.

