

# Ubiquitous Monitoring for Industrial Cyber-Physical Systems over Relay Assisted Wireless Sensor Networks

CAILIAN CHEN\*, (*Member, IEEE*), JING YAN†, NING LU‡, (*Student Member, IEEE*),  
YIYIN WANG\*, (*Member, IEEE*), XIAN YANG†, XINPING GUAN\*, (*Senior Member, IEEE*)

\*Department of Automation, Shanghai Jiao Tong University, Shanghai, P. R. China;  
Key Laboratory of System Control and Information Processing, Ministry of Education of China;  
Cyber Joint Innovation Center, Hangzhou, China.

†Institute of Electrical Engineering, Yanshan University, Qinhuangdao, P. R. China

‡Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, Ontario, Canada  
CORRESPONDING AUTHOR: C. CHEN (cailianchen@sjtu.edu.cn)

The work was partially supported by NSF of China under U1405251, 61273181, 61221003, 61290322 and 61174127, by Ministry of Education of China under NCET-13-0358 and 20110073120025, by Science and Technology Commission of Shanghai Municipality (STCSM), China under 13QA1401900, and by Yanshan University under B832 and 14LGA010.

**Abstract**—Ubiquitous monitoring over wireless sensor networks (WSNs) is of increasing interest in industrial Cyber-Physical Systems (CPSs). Question of how to understand a situation of physical system by estimating process parameters is largely unexplored. This paper is concerned with the distributed estimation problem for industrial automation over relay assisted wireless sensor networks (WSNs). Different from most existing works on WSN with homogeneous sensor nodes, the network considered in this paper consists of two types of nodes, i.e., sensing nodes (SNs) which is capable of sensing and computing, and relay nodes (RNs) which is only capable of simple data aggregation. We first adopt a Kalman filtering (KF) approach to estimate the unknown physical parameters. In order to facilitate the decentralized implementation of the KF algorithm in relay assisted WSNs, a tree-based broadcasting strategy is provided for distributed sensor fusion. With the fused information, consensus-based estimation algorithms are proposed for SNs and RNs, respectively. The proposed method is applied to estimate the slab temperature distribution in a hot rolling process monitoring system which is a typical industrial CPS. It is demonstrated that the introduction of RNs improves temperature estimation efficiency and accuracy compared with the homogeneous WSN with SNs only.

**Index Terms**—Cyber-Physical System; Ubiquitous monitoring; Wireless sensor network; Relay; Consensus; Estimation



## 1 INTRODUCTION

CYBER-PHYSICAL Systems (CPS) refers to a new generation of complex systems integrating physical processes, ubiquitous computation, efficient communication and effective control. One of typical example of CPS is process control system in industrial automation [1], [2]. Being able to quickly and cost effectively monitor the system process is essential for industrial automation. Whatever the specifics of process monitored, all industrial systems share the critical requirement, i.e. timely situation awareness with sensing, data processing and communication. Wireless Sensor Network (WSN) is one of the key technologies associated with ubiquitous monitoring for CPSs. WSNs are formed by small-sized, low cost, and wireless-communication capable sensors, which have demonstrated a great potential for many practical applications in industrial automation. Specifically, by using industrial wireless technology, a wealth of process data such as temperature, humidity, pressure, viscosity and vibration intensity measurements can be collected through sensing units and transferred for operation and management [3]–[5]. Compared with tradition-

al wired industrial monitoring and control systems, the collaborative nature of WSNs brings several advantages, including self-organization, rapid deployment, flexibility, and inherent intelligent-processing capability. Therefore, WSN plays a vital role in creating a highly reliable and self-healing industrial CPS that rapidly responds to real-time events with appropriate actions [6], [7].

However, there exists a gap between the wealth of distributed information captured and the understanding of a situation of physical systems. Distributed estimation over WSN is a key process bridging this gap by locally carrying out computation and transmitting only the required and/or partially processed data. An important issue here is, how to ensure the estimations of sensor nodes reach consensus and converge to the true values of the physical parameters.

On the aspect of distributed consensus estimation, Olfati-Saber first introduced the consensus strategy for the distributed estimation problem [8]. Motivated by the consensus strategy, alternative consensus-based distributed Kalman filters are designed in [9], [10]. The filters are composed of two stages, i.e., the Kalman-like measurement update and the estimate fusion using a

consensus matrix. Moreover, consensus-based estimation algorithms has been proposed for process monitoring and control, e.g., [11] and references therein. In order to improve the autonomy estimation ability of sensors, mobile WSNs is considered in [12], [14]. However, the harsh industrial environment introduces several challenging issues for mobile WSNs as summarized below: 1) Constrained by the limited physical size, industrial sensor nodes have limited battery energy supply; 2) For the sake of safety, sensors in industrial automation are usually required to be deployed with proper locations to avoid the corrosive environments, and strong humidity, vibrations, dirt and dust or other improper conditions. These challenging issues limit the applications of mobile WSNs in industrial automation. Thus, an interesting question is how to design a virtual mobility strategy for static WSNs to improve the autonomy estimation ability. This paper provides an example.

On the other hand, most of the existing research focuses on the homogeneous WSNs, i.e., all sensor nodes possess identical communication and computation capabilities, and their roles in distributed estimation are homogeneous. Recently, it is reported that the heterogeneity in the sensors' roles can improve the communication ability, prolong the network lifetime and increase the communication reliability [15], [16]. Question of how to improve the distributed estimation capability of heterogeneous network is largely unexplored. Recently, we consider a class of heterogeneous network which consists of a large number of cheap and low-end nodes only with the function of communication and simple data aggregation, and a small number of high-quality sensors with powerful functions of sensing, communication and computation. The estimation capability of this kind of heterogeneous WSNs has been demonstrated in [17]–[19] to be enhanced compared with the homogeneous WSNs.

Inspired by the aforementioned considerations, the relay assisted WSN is investigated for distributed estimation in industrial CPS. The network consists of two types of nodes, i.e., sensing nodes (SNs) which are capable of sensing and computing, and relay nodes (RNs) which are only capable of simple data aggregation. A Kalman filtering (KF) approach is first applied to estimate the unknown physical parameters. A Tree-based broadcasting communication is provided for distributed sensor fusion to facilitate the decentralized implementation of the KF in WSNs. Based on the fused information, we propose consensus-based estimation algorithms for SNs and RNs, respectively. To improve the autonomy estimation ability, a dynamic duty-cycle scheduling mechanism is designed for SNs according to the estimate, such that the network lifetime can be prolonged. Finally, we apply the proposed method to the slab temperature estimation in a hot rolling process. Simulation results demonstrate that the introduction of RNs improves the temperature estimation efficiency and accuracy compared with the homogeneous WSN with SNs only.

The remainder of this paper is organized as follows.

Section II gives the problem formulation. In Section III, the main results of distributed parameter estimation are presented for two types of nodes in the network. An application of distributed estimation in hot rolling process monitoring is given in Section IV, followed with the conclusion in Section V.

## 2 PROBLEM FORMULATION

Consider an industrial CPS with the following process dynamics

$$q(k+1) = F(k)q(k) + \omega(k), \quad (1)$$

where  $q = [q_1^T, q_2^T, \dots, q_m^T]^T$  is the state vector of the physical system (1) with  $q_i = [q_{1,i}, q_{2,i}, \dots, q_{n,i}]^T$ ,  $i = 1, 2, \dots, m$ , and  $m$  and  $n$  are positive integers. The matrix  $F \in \mathbb{R}^{mn \times mn}$ , and  $\omega \in \mathbb{R}^{mn}$  is the disturbance assumed to be zero-mean and white. The initial state of  $q$ , denoted as  $q_0$ , is a Gaussian random variable with the known mean  $\mathbb{E}\{q_0\} = \bar{q}_0$  and positive definite covariance  $\mathbb{E}\{(q_0 - \bar{q}_0)(q_0 - \bar{q}_0)^T\} = \Pi_0$ . Table 1 shows the main notations used in this paper.

TABLE 1  
Notations

|                           |   |
|---------------------------|---|
| $\mathbb{R}^n$            | $n$ -dimensional Euclidean space                  |
| $\mathbb{R}^{n \times n}$ | Set of $n \times n$ real valued matrices          |
| $I$                       | Identity matrix with appropriate dimensions       |
| $\text{diag}\{\cdot\}$    | Diagonal block matrix                             |
| $\mathbf{1}$              | Vector of ones                                    |
| $\text{tr}(\cdot)$        | Trace of a matrix                                 |
| $\mathbf{0}$              | A vector or a matrix of zeros                     |
| $\lambda_{\max}(\cdot)$   | Maximum eigenvalue of a symmetric matrix          |
| $\lambda_{\min}(\cdot)$   | Minimum eigenvalue of a symmetric matrix          |
| $\mathbb{E}\{x\}$         | Mathematical expectation of a random variable $x$ |
| $\otimes$                 | Kronecker product                                 |
| $q$                       | Parameter vector to be estimated                  |
| $\hat{q}_i$               | Estimate of the unknown parameter $q$             |
| $\bar{q}_i$               | Prior estimate (or receding) of the state $q$     |

Consider the monitoring problem for the physical system (1) by the means of parameter estimation in a relay assisted WSN. The network is composed of  $N$  nodes with  $M$  number of SNs capable of target parameter estimation and  $N - M$  number of RNs capable of data aggregation only, where  $N$  and  $M$  are positive integers and  $M < N$ . The index sets of SNs and RNs in the network are denoted by  $\mathcal{I}_S = \{1, 2, \dots, M\}$  and  $\mathcal{I}_R = \{M+1, M+2, \dots, N\}$ , respectively. For any SN  $i$ , it can sense the unknown physical parameter vector  $q$  at its fixed location  $p_i \in \mathbb{R}^2$ , and the measurement  $z_i \in \mathbb{R}$  is given by

$$z_i(k) = H_i(k)q(k) + \epsilon_i(k) \quad (2)$$

where  $\epsilon_i \in \mathbb{R}$  is white Gaussian measurement noise,  $H_i = \Psi(p_i) = [(\psi^{(1)})^T, (\psi^{(2)})^T, \dots, (\psi^{(m)})^T]$  is measurement matrix, and  $\psi^{(j)} = [\psi_{1,j}, \psi_{2,j}, \dots, \psi_{n,j}]^T, \forall j = 1, \dots, m$  are model basis functions related to the location  $p_i$  of SN  $i$ .

As for RNs, their role is to aggregate the data received from neighboring nodes and relay the aggregation to

neighboring nodes. So they cannot measure the parameters directly because they are not imbedded with sensing modules. The reliable wireless communications between nodes can only be ensured within a communication range  $r \in \mathbb{R}^+$ .

For SNs and RNs, the communication topology is modeled as a weighted directed graph  $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ , where  $\mathcal{V} = \mathcal{I}_S \cup \mathcal{I}_R = \{1, 2, \dots, N\}$  is the vertex set indicating SNs and RNs,  $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$  is the set of communication links over which local information is exchanged, and  $\mathcal{A} = [a_{ij}]$  is the weight matrix associated with  $\mathcal{E}$ . Since the graph is directed,  $\mathcal{A}$  may be asymmetric. Directed edge  $(i, j) \in \mathcal{E}$  means that the information flows from the vertex  $i$  to the vertex  $j$ . The set  $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}, j \neq i\}$  is the neighbor set of vertex  $i$  in  $\mathcal{G}$ . Graph  $\mathcal{G}$  is connected if there is a path connecting every pair of vertices.

Given the above physical dynamics model and network model, the parameter estimation problem for CPS monitoring can now be stated as follows.

*Problem (Distributed Estimation):* Consider the distributed estimation problem by the given relay assisted WSNs for the physical system (1) to achieve the following two objectives:

- SNs can estimate the parameters based on a consensus based estimation method in a distributed manner;
- Based on the estimate of SNs and aggregation information of RNs, a dynamic duty-cycle scheduling mechanism is to be designed for SNs such that they are of autonomy estimation ability.

### 3 CONSENSUS BASED DISTRIBUTED ESTIMATION OVER RELAY ASSISTED WSNs

In this section, we first present a standard Kalman filter to estimate the unknown physical parameters. A distributed Tree-based broadcasting communication algorithm and a consensus based distributed estimation method are then designed. Furthermore, a dynamic duty-cycle scheduling mechanism is designed to improve the estimation capability.

#### 3.1 Standard Kalman Filter

For SN  $i$ , given the past information  $Z_i(k) = \{z_i(0), z_i(1), \dots, z_i(k)\}$  by the measurements of the parameters up to time  $k$ , the estimates of the state vector  $q$  are obtained as follows:

$$\begin{aligned}\hat{q}_i(k) &= \mathbb{E}\{q(k) | Z_i(k)\}, \\ \bar{q}_i(k) &= \mathbb{E}\{q(k) | Z_i(k-1)\}.\end{aligned}\quad (3)$$

Let  $\eta_i = \hat{q}_i - q$  and  $\bar{\eta}_i = \bar{q}_i - q$  be the estimate errors. Then, the error covariance matrices associated with the estimates  $\hat{q}_i$  and  $\bar{q}_i$  are respectively given by

$$\begin{aligned}M_i(k) &= \mathbb{E}\{\eta_i(k)\eta_i(k)^T\}, \\ P_i(k) &= \mathbb{E}\{\bar{\eta}_i(k)\bar{\eta}_i(k)^T\}.\end{aligned}\quad (4)$$

---

#### Algorithm 1: Distributed Kalman filter algorithm over relay assisted WSNs

---

- 1: Initialization:  $P_i = M\Pi_0, \bar{q}_i = q_0$
  - 2: **while** new data exists **do**
  - 3: Estimate the sums of  $y(k)$  and  $S(k)$  by consensus-based distributed algorithm, as provided in Subsection 3.2
  - 4: Estimate the state  $q$  using standard Kalman filter:  
 $M_i(k) = (P_i^{-1}(k) + S(k))^{-1}$   
 $\hat{q}_i(k) = \bar{q}_i(k) + M_i(k)(y(k) - S(k)\bar{x}_i(k)) + \sum_{j \in \mathcal{N}_i} (\bar{q}_j(k) - \bar{q}_i(k)),$
  - 5: Update the receding states:  
 $P_i(k+1) \leftarrow F M_i(k) F^T + Q(k)$   
 $\bar{q}_i(k+1) \leftarrow F \hat{q}_i(k)$
  - 6: Adopt a scheduling mechanism, as provided in Subsection 3.3
  - 7: **end while**
  - 8: Return  $\hat{q}_i(k)$
- 

With the definitions of (3) and (4), the following lemma gives a detailed iteration process of Kalman filtering.

*Lemma 1:* (Kalman Filter [12], [21]) Consider the relay assisted WSN with linear perception model (2) for the unknown dynamic physical parameters (1). Let  $R_i$  and  $Q$  denote the covariance matrices of  $\epsilon_i$  and  $\omega$ , respectively. Assume that  $\epsilon_i$  is uncorrelated. Suppose each SN computes two central sums  $y = \sum_{i=1}^M H_i^T R_i^{-1} z_i = \sum_{i=1}^M \Psi^T(p_i) R_i^{-1} z_i$  and  $S = \sum_{i=1}^M H_i^T R_i^{-1} H_i = \sum_{i=1}^M \Psi^T(p_i) R_i^{-1} \Psi(p_i)$  and applies the following local estimation iterations

$$\begin{aligned}M_i(k) &= (P_i^{-1}(k) + S(k))^{-1}, \\ \hat{q}_i(k) &= \bar{q}_i(k) + M_i(k)(y(k) - S(k)\bar{q}_i(k)) \\ &\quad + \sum_{j \in \mathcal{N}_i} (\bar{q}_j(k) - \bar{q}_i(k)), \\ P_i(k+1) &= F(k) M_i(k) F^T + Q(k), \\ \bar{q}_i(k+1) &= F(k) \hat{q}_i(k).\end{aligned}\quad (5)$$

Then, the state estimates of all SNs can converge to a consensus, i.e.,  $\hat{q}_1 = \hat{q}_2 = \dots = \hat{q}_M$ .

It is noted that Kalman filter design is a centralized scheme since a fusion center is required to compute the sums of  $y$  and  $S$ . However, the centralized scheme is not applicable for WSNs due to its costly data transmission and energy limitation. One possible solution for this problem is to approximate the averages  $y$  and  $S$ , so that the distributed Kalman filtering algorithm can be proposed. Inspired by this consideration, we give our main results in Subsection 3.2 to compute the averages  $y$  and  $S$ . After we obtain the estimation by SNs and RNs, a dynamic duty-cycle scheduling mechanism is designed in Subsection 3.3 to improve the estimation capability. Integrating the proposed consensus-based estimation method and duty-cycle scheduling mechanism, we propose a distributed Kalman filtering algorithm for the relay assisted WSN as shown in Algorithm 1.

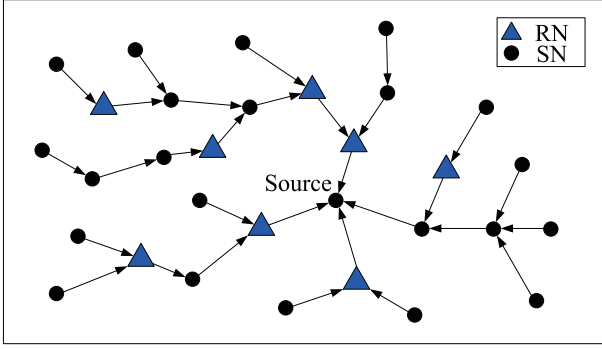


Fig. 1. Example of the tree construction in relay assisted WSNs.

---

**Algorithm 2:** Distributed Tree-based broadcasting communication

---

- 1: Initiation:  $\phi_i = [H_i^T R_i^{-1} z_i, H_i^T R_i^{-1} H_i]$  for SN  $i \in \mathcal{I}_S$ ;  
 $\phi_i = 0$  for RN  $i \in \mathcal{I}_R$
  - 2: **while** new date exists **do**
  - 3:   **for** all leaf nodes in tree **do**
  - 4:     Transmit  $\phi_i$  to its neighbors
  - 5:   **end for**
  - 6:   **for** all intermediate nodes receiving  $\phi_i$  from their neighbors **do**
  - 7:      $\phi_j \leftarrow \phi_j + \sum_{j \in \mathcal{N}_i} \phi_i$
  - 8:     Transmit  $\phi_j$  to its parent
  - 9:   **end for**
  - 10: **end while**
  - 11: Output:  $\phi_1$
  - 12: SN 1 broadcasts  $\varphi = \phi_1/M$  to its children
- 

### 3.2 Distributed Tree-based broadcasting communication and Consensus-based Estimation Algorithm

Since each SN needs to estimate the sums of  $y$  and  $S$ , we divide the estimation into two phases, i.e. *PRE-ESTIMATION* and *ESTIMATION* phases. In the *PRE-ESTIMATION* phase,  $y$  and  $S$  can be approximated through a distributed Tree-based broadcasting communication. In order to attenuate the impact of measurement noise on the estimate in WSNs [22], [23], we present a consensus-based distributed estimation method in the *ESTIMATION* phase.

In the *PRE-ESTIMATION* phase, represent SN  $i$ 's contribution (i.e.,  $H_i^T R_i^{-1} z_i$  and  $H_i^T R_i^{-1} H_i$ ) for the sums of  $y$  and  $S$  by the vector  $\phi_i$ , which is correlated with  $H_i$ ,  $R_i$ , and  $z_i$ . For simplicity, assume that SN 1 is the approximator of the sums of  $y$  and  $S$ . In this paper, a tree topology (see Fig. 1) is constructed by using the TreeCast Algorithm [24], [25]. Under the constructed tree, all leaf and intermediate nodes make information fusion to transmit an approximate result of  $y$  and  $S$  to SN 1. Subsequently, SN 1 broadcasts this result to all nodes through direct or indirect ways. The Tree-based broadcasting communication can be easily implemented by Algorithm 2.

*Remark 1:* In the Tree-based broadcasting communication, any node  $i$  ( $i \in \mathcal{V}/\{1\}$ ) transmits the data only to its neighbors. Thus, aside from the source node, all nodes form a distributed communication system. For the source node, we adopt a broadcast operation, where the source broadcasts the message  $\varphi$  to its children. According to Zigbee or other protocol specifications, this broadcast operation is technically feasible, and the similar broadcast operation can be found in Refs. [26], [27].

In the *ESTIMATION* phase, each sensor  $i$  receives an approximation average of  $y$  and  $S$  from the approximator node, denoted by  $\tilde{\varphi}_i$ . Take the communication noise into account,  $\tilde{\varphi}_i$  is rewritten into

$$\tilde{\varphi}_i(k) = \varphi + \vartheta_i(k), \forall i \in \mathcal{I}_S, \quad (6)$$

where  $\varphi$  is the accurate but unknown average of  $y$  and  $S$ , and  $\vartheta_i$  is white Gaussian noise. Let  $\vartheta = [\mathbf{0}, \vartheta_2^T, \dots, \vartheta_M^T]^T$  with bounded covariance.

To reduce the effect of noise on the estimation, a consensus-based distributed estimation method is designed as follows.

For SN  $i$  ( $i \in \mathcal{I}_S$ ),

$$\zeta_i(k+1) = \zeta_i(k) + \varepsilon \rho(k) (\tilde{\varphi}_i(k) - \zeta_i(k)) + \beta \rho(k) \sum_{j \in \mathcal{N}_i} a_{ij} (\zeta_j(k) - \zeta_i(k)), \quad (7)$$

and for RN  $i$  ( $i \in \mathcal{I}_R$ ),

$$\zeta_i(k) = \sum_{j \in \mathcal{N}_i} \gamma_{ij} \zeta_j(k), \quad (8)$$

where  $\zeta_i$  is the local estimation of  $\varphi$ ,  $\rho \in \mathbb{R}^+$  is the scale governing the update rate of the information,  $\varepsilon, \beta \in \mathbb{R}^+$  are estimator gains, and  $\gamma_{ij} \in \mathbb{R}^+$  denotes the weight satisfying  $\sum_{j \in \mathcal{N}_i} \gamma_{ij} = 1$ . The coefficient  $a_{ij} = 1$  if sensor  $i$  can receive signal from node  $j$ , otherwise  $a_{ij} = 0$ .

It is worth to note that the estimation algorithms in (7) and (8) for SNs and RNs are different and the estimation approaches for homogeneous sensor network cannot be used directly here. Thus, we use a graph transformation operation proposed in our previous works [17]–[19] to rearrange (7) and (8). The graph transformation operation is given as follows.

*Lemma 2:* [19] If the graph  $\mathcal{G}$  is connected, the state of each node in  $\mathcal{I}_R$  can be expressed as a convex combination of the states of nodes in  $\tilde{\mathcal{I}}_S$ , i.e.,

$$\zeta_i(k) = \sum_{j \in \tilde{\mathcal{I}}_S} \tilde{\gamma}_{ij} \zeta_j(k), \forall i \in \mathcal{I}_R, \quad (9)$$

where  $\tilde{\mathcal{I}}_S = \{j \in \mathcal{I}_S : \exists i \in \mathcal{I}_R, (j, i) \in \mathcal{E}\}$ . It is easily seen that  $\sum_{j \in \tilde{\mathcal{I}}_S} \tilde{\gamma}_{ij} = 1$ ,  $0 \leq \tilde{\gamma}_{ij} \leq 1$  if  $j \in \tilde{\mathcal{I}}_S \cap \mathcal{N}_i$ , and otherwise  $\tilde{\gamma}_{ij} = 0$ .

Define the SN and RN neighbors of the sensor  $i$  as  $\mathcal{N}_i^S = \mathcal{N}_i \cap \mathcal{I}_S$  and  $\mathcal{N}_i^R = \mathcal{N}_i \cap \mathcal{I}_R$ , respectively, where  $\mathcal{N}_i = \mathcal{N}_i^S \cup \mathcal{N}_i^R$ . For SN  $i \in \mathcal{I}_S$ , rearranging (7) based on

Lemma 2, we have

$$\begin{aligned} & \zeta_i(k+1) \\ = & \zeta_i(k) + \beta\rho(k) \left[ \sum_{j \in \mathcal{N}_i^S} a_{ij}(\zeta_j(k) - \zeta_i(k)) \right. \\ & \left. + \sum_{j \in \mathcal{I}_S \setminus \{i\}} \sum_{k \in \mathcal{N}_i^R} a_{ik} \tilde{\gamma}_{kj}(\zeta_j(k) - \zeta_i(k)) \right] \\ & + \zeta_i(k) + \varepsilon\rho(k)\tilde{\varphi}_i(k) - \varepsilon\rho(k)\zeta_i(k). \end{aligned} \quad (10)$$

In the following, we analyze the unbiasedness and consistency of the estimation algorithm. Similar to the assumption in [14], let  $\varphi$  and  $\tilde{\varphi}_i$  for each  $\zeta_i$  be scalars for simplicity in notation, i.e.,  $\varphi, \tilde{\varphi}_i \in \mathfrak{R}$ . Then, we can rewrite (10) into a compact form

$$\zeta(k+1) = \Gamma(k)\zeta(k) + \varepsilon\rho(k)\tilde{\varphi}(k), \quad (11)$$

where  $\zeta = [\zeta_1, \zeta_2, \dots, \zeta_M]^T$ ,  $\tilde{\varphi} = [\tilde{\varphi}_1, \tilde{\varphi}_2, \dots, \tilde{\varphi}_M]^T$ ,  $\Gamma(k) = I - \varepsilon\rho(k)I - \beta\rho(k)L$ ,  $L = [L_{ij}] \in \mathfrak{R}^{M \times M}$  with entries

$$L_{ij} = \begin{cases} \sum_{j \in \mathcal{N}_i^S} a_{ij} + \sum_{j \in \mathcal{N}_i^R} a_{ij} \sum_{k \in \mathcal{I}_S \setminus \{i\}} \tilde{\gamma}_{jk}, & j = i, \\ -a_{ij} - \sum_{k \in \mathcal{N}_i^R} a_{ik} \tilde{\gamma}_{kj}, & j \neq i. \end{cases}$$

With the notations, the following lemma is given.

*Lemma 3:* [19] If the graph  $\mathcal{G}$  is connected, then zero is a simple eigenvalue of  $L$  and the corresponding left eigenvector  $[\xi_1, \xi_2, \dots, \xi_M]^T$  is positive.

Define the error variable

$$e_i(k) = \zeta_i(k) - \varphi, \quad \forall i \in \mathcal{I}_S. \quad (12)$$

Based on Lemma 3, one has  $L\mathbf{1} = \mathbf{0}$ . Rearranging (11) and noting that  $L\mathbf{1} = \mathbf{0}$ , we can rewrite (12) into the following compact form

$$e(k+1) = \Gamma(k)e(k) + \varepsilon\rho(k)\vartheta(k), \quad (13)$$

where  $e = [e_1, e_2, \dots, e_M]^T$  and  $\vartheta = [0, \vartheta_2, \dots, \vartheta_M]^T$ .

*Theorem 1:* (Asymptotic Unbiasedness) If the directed graph  $\mathcal{G}$  is connected, and  $\rho(k)$  satisfies

$$\sum_{k=1}^{\infty} \rho(k) = \infty, \quad \sum_{k=1}^{\infty} \rho^2(k) < \infty \quad (14)$$

then  $\zeta_i(k)$  is asymptotically unbiased for  $i \in \mathcal{I}_S$ , i.e.,  $\lim_{k \rightarrow \infty} \mathbb{E}\{\zeta_i(k)\} = 0, \forall i \in \mathcal{I}_S$ .

**Proof.** Define  $\Theta = \text{diag}\{\xi_1, \xi_2, \dots, \xi_M\}$ ,  $D = \sqrt{\Theta/\beta}$ , and  $\tilde{e}(k) = De(k)$ . This yields

$$\tilde{e}(k+1) = D\Gamma(k)e(k) + \varepsilon\rho(k)\vartheta(k). \quad (15)$$

Taking expectation on both side of (15) and noting that  $\mathbb{E}\{\vartheta(k)\} = 0$ , we obtain

$$\mathbb{E}\{\tilde{e}(k+1)\} = D\Gamma(k)D^{-1}\mathbb{E}\{\tilde{e}(k)\}. \quad (16)$$

Continue the above recursion, and we see that for  $k > k_0$

$$\mathbb{E}\{\tilde{e}(k)\} = \prod_{s=k_0}^{k-1} D\Gamma(s)D^{-1}\mathbb{E}\{\tilde{e}(k_0)\}, \quad (17)$$

and therefore, for  $\forall k > k_0$

$$\|\mathbb{E}\{\tilde{e}(k)\}\| \leq \prod_{s=k_0}^{k-1} \|D\Gamma(s)D^{-1}\| \|\mathbb{E}\{\tilde{e}(k_0)\}\|. \quad (18)$$

Defining  $E = \varepsilon I + D(\beta L)D^{-1} + D^{-1}(\beta L^T)D$ , we see that  $E = \varepsilon I + D^{-1}(\Theta L + L^T \Theta)D^{-1}$ . We also define  $\Omega = \varepsilon I + \beta L$ ,  $\Omega_1 = D^{-1}\Omega^T D^2 \Omega D^{-1}$ ,  $\hat{E} = \varepsilon I + E$ ,  $\Omega_2 = \hat{E} - \rho(k)\Omega_1$ . As  $D^{-1}(\varepsilon I)D = D(\varepsilon I)D^{-1} = \varepsilon I$ , then  $D^{-1}\Omega^T D + D\Omega D^{-1} = 2\varepsilon I + D^{-1}(\beta L^T)D + D(\beta L)D^{-1} = \varepsilon I + E = \hat{E}$ .

Because  $\Gamma(k) = I - \rho(k)\Omega$ , we have

$$\begin{aligned} D^{-1}\Gamma^T(k)D^2\Gamma(k)D^{-1} &= I - \rho(k)\hat{E} + \rho^2(k)\Omega_1 \\ &= I - \rho(k)\Omega_2. \end{aligned} \quad (19)$$

Note that  $\Theta L + L^T \Theta$  is positive definite, then  $E$  is positive definite. Moreover, we also see that  $\hat{E}$  is positive definite, and  $\Omega_1$  is positive semidefinite. Therefore,  $\lambda_{\min}(\hat{E}/2) > 0$  and  $\lambda_{\max}(\Omega_1) > 0$ . If (14) is satisfied, we can obtain  $\rho(k) \rightarrow 0$  as  $k \rightarrow \infty$ . It now follows that: there exists  $k_1$  such that  $\rho(k)\lambda_{\max}(\Omega_1) \leq \lambda_{\min}(\hat{E}/2)$  for all  $k \geq k_1$ . Specially, using Rayleigh-Ritz theorem [14], we obtain that for  $k \geq k_1$

$$\lambda_{\min}(\hat{E}/2) \leq \lambda_{\min}(\Omega_2) \leq \lambda_{\max}(\Omega_2) \leq \lambda_{\max}(\hat{E}). \quad (20)$$

In particular, (20) and the fact  $\rho(k) \rightarrow 0$  with  $k \rightarrow \infty$  imply the existence of a scalar constant  $k_2 > k_1$  such that

$$\rho(k) \leq \frac{1}{\lambda_{\max}(\hat{E})}, \quad \forall k \geq k_2. \quad (21)$$

Furthermore, (19) and (20) imply

$$\begin{aligned} & \|D\Gamma(k)D^{-1}\|^2 \\ &= \|D^{-1}\Gamma^T(k)D^2\Gamma(k)D^{-1}\| \\ &= 1 - \rho(k)\lambda_{\min}(\Omega_2), \quad \forall k \geq k_2. \end{aligned} \quad (22)$$

Making  $k_0 = k_2$  and submitting (22) to (18), we have

$$\begin{aligned} & \|\mathbb{E}\{\tilde{e}(k)\}\| \\ & \leq \prod_{s=k_0}^{k-1} \sqrt{1 - \rho(s)\lambda_{\min}(\Omega_2)} \|\mathbb{E}\{\tilde{e}(k_0)\}\|. \end{aligned} \quad (23)$$

It is clear that  $1 \geq \rho(k)\lambda_{\min}(\Omega_2) \geq 0$  from (20) and (21). Therefore,  $\ln(1 - \rho(k)\lambda_{\min}(\Omega_2)) \leq -\rho(k)\lambda_{\min}(\Omega_2)$  holds. Furthermore, (20) and (23) imply that for  $k \geq k_0$

$$\begin{aligned} & \|\mathbb{E}\{\tilde{e}(k)\}\| \\ & \leq \exp\left[-\frac{1}{2} \prod_{s=k_0}^{k-1} \rho(s)\lambda_{\min}(\Omega_2)\right] \|\mathbb{E}\{\tilde{e}(k_0)\}\| \\ & \leq \exp\left[-\frac{1}{2} \prod_{s=k_0}^{k-1} \lambda_{\min}(\hat{E}/2)\right] \|\mathbb{E}\{\tilde{e}(k_0)\}\| \prod_{s=k_0}^{k-1} \rho(s) \end{aligned}$$

Note (14) and  $\lambda_{\min}(\hat{E}/2) > 0$ , then we have  $\lim_{k \rightarrow \infty} \|\mathbb{E}\{\tilde{e}(k)\}\| = \lim_{k \rightarrow \infty} \left\| \sqrt{\Theta/\beta} \mathbb{E}\{e(k)\} \right\| = 0$ . As  $\left\| \sqrt{\Theta/\beta} \mathbb{E}\{e(k)\} \right\| \geq \min_{1 \leq i \leq M} \sqrt{\xi_i/\beta}$  and  $\xi_i/\beta > 0$ . Thus, we obtain  $\|\mathbb{E}\{e(k)\}\| \rightarrow 0$  with  $k \rightarrow \infty$ . ■

Next, we analyze the consistency of the consensus-based distributed estimation algorithm. The proof relies on a well known Robbins-Siegmund Lemma on stochastic processes as follows.

*Lemma 4:* [28] Let  $\{\mathcal{F}_k\}_k \geq 0$  denote a sequence of  $\sigma$ -algebras and  $V(k)$ ,  $\mu(k)$ ,  $\kappa(k)$ , and  $\varkappa(k)$  denote  $\mathcal{F}_k$ -measurable nonnegative random variables such that for all  $k \geq 0$ ,  $\mathbb{E}\{V(k+1)|\mathcal{F}_k\}$  exists and

$$\mathbb{E}\{V(k+1)|\mathcal{F}_k\} \leq (1 + \mu(k))V(k) + \kappa(k) - \varkappa(k) \text{ a.s.},$$

with  $\sum_{k=0}^{\infty} \mu(k) < \infty$  and  $\sum_{k=0}^{\infty} \kappa(k) < \infty$  almost surely. Then, there exists a nonnegative random variable  $\kappa^*$  such that

$$\mathbb{P}\{\lim_{k \rightarrow \infty} V(k) = \kappa^*\} = 1, \text{ and } \sum_{k=0}^{\infty} \varkappa(k) < \infty.$$

*Theorem 2:* (Consistency) If the directed graph  $\mathcal{G}$  is connected, and  $\rho(k)$  satisfies the conditions in (14), then the estimator of each SN  $i$  is consistent, i.e.,  $\mathbb{P}\{\lim_{k \rightarrow \infty} \zeta_i(k) = \varphi\} = 1, \forall i \in \mathcal{I}_S$ .

**Proof.** Defining the error process  $V(e) = \sum_{k=0}^{\infty} \xi_i \|e_i\| / \beta$ , we have  $V(e) = \tilde{e}^T(k)\tilde{e}(k)$ . We also define the  $\sigma$ -algebras  $\mathcal{F}_k = \sigma\{\vartheta(0), \vartheta(1), \dots, \vartheta(k-1)\}$ . Then,  $\vartheta(k)$  is independent of  $\mathcal{F}_k$  and  $\mathbb{E}\{\vartheta(k)|\mathcal{F}_k\} = \mathbb{E}\{\vartheta(k)\} = 0$ .

From (12) and (13), we take expectation conditioned on  $\mathcal{F}_k$  on both side of the expression of  $V(e(k+1))$ . This yields

$$\begin{aligned} & \mathbb{E}\{V(e(k+1))|\mathcal{F}_k\} \\ &= e^T(k)\Gamma^T(k)D^2\Gamma(k)e(k) + \varepsilon^2\rho^2(k)\vartheta^T(k)\vartheta(k) \\ &= \tilde{e}^T(k)D^{-1}\Gamma^T(k)D^2\Gamma(k)D^{-1}e(k) \\ & \quad + \varepsilon^2\rho^2(k)\vartheta^T(k)\vartheta(k). \end{aligned} \quad (24)$$

Next, based on (19) and (24), we have

$$\begin{aligned} & \mathbb{E}\{V(e(k+1))|\mathcal{F}_k\} \\ &= V(e(k)) - \rho(k)\tilde{e}^T(k)\hat{E}e(k) \\ & \quad + \rho^2(k)\tilde{e}^T(k)\Omega_1\tilde{e}(k) + \varepsilon^2\rho^2(k)\vartheta^T(k)\vartheta(k). \end{aligned} \quad (25)$$

It is clear that  $\hat{E}$  is positive definite, which implies

$$\begin{aligned} & \mathbb{E}\{V(e(k+1))|\mathcal{F}_k\} \\ & \leq V(e(k)) - \rho(k)\lambda_{\min}(\hat{E})V(e(k)) \\ & \quad + \rho^2(k)\lambda_{\max}(\Omega_1)V(e(k)) + \varepsilon^2\rho^2(k)\text{tr}[\vartheta^T(k)\vartheta(k)]. \end{aligned}$$

Again defining  $\mu(k) = \rho^2(k)\lambda_{\max}(\Omega_1)$ ,  $\kappa(k) = \varepsilon^2\rho^2(k)\text{tr}[\vartheta^T(k)\vartheta(k)]$  and  $\varkappa(k) = \rho(k)\lambda_{\min}(\hat{E})V(e(k))$ , we obtain that:  $\mu(k)$ ,  $\kappa(k)$  and  $\varkappa(k)$  are all nonnegative.

Suppose that (14) holds, and then we have  $\sum_{k=0}^{\infty} \mu(k) < \infty$  and  $\sum_{k=0}^{\infty} \kappa(k) < \infty$ . Specially, applying Lemma 4 to  $V(e)$  implies the existence of a random variables  $\kappa^* \geq 0$  such that

$$\begin{cases} \mathbb{P}\{\lim_{k \rightarrow \infty} V(k) = \kappa^*\} = 1 \\ \sum_{k=0}^{\infty} \varkappa(k) < \infty. \end{cases} \quad (26)$$

Since  $\lambda_{\min}(\hat{E}) > 0$ , we also have

$$\sum_{k=0}^{\infty} \rho(k)V(e(k)) < \infty. \quad (27)$$

Suppose that there exists a subsequence  $\{k_1, k_2, \dots\} \subset \{1, 2, \dots\}$  such that

$$\mathbb{P}\{\lim_{k \rightarrow \infty} V(k) = 0\} = 1. \quad (28)$$

---

### Algorithm 3: Dynamic duty-cycle Scheduling Mechanism

---

- 1: Initialization: Awake SN  $i$  with its position  $p_i(k+1)$
  - 2: **while** new data exists **do**
  - 3:   Update the virtual position  $p_i(k+1)$
  - 4:   **if**  $\min_{j \in \mathcal{B}_i} \|p_i - p_j\| \leq \mu_{\#}$  **then**
  - 5:     SN  $i$  and  $j$  turn into sleeping and awake modes, respectively
  - 6:   **else**
  - 7:     Keeps in an existing state
  - 8:   **end if**
  - 9: **end while**
- 

Assume otherwise that there is a constant  $\bar{\kappa}^* > 0$  so that  $\mathbb{P}\{\mathcal{C}\} \geq \bar{\kappa}^*$ , where  $\mathcal{C} = \{\vartheta : \liminf_{k \rightarrow \infty} V(e(k, \vartheta)) \geq 2\bar{\kappa}^*\}$ . For any  $\vartheta \in \mathcal{C}$ , we can find a random variables  $k^*(\bar{\kappa}^*, \vartheta) \geq 0$  such that  $V(e(k, \vartheta)) \geq \bar{\kappa}^*$ , whenever  $k \geq k^*$ . Hence we obtain

$$\sum_{k=0}^{\infty} \rho(k)V(e(k)) \geq \bar{\kappa}^* \sum_{k=0}^{\infty} \rho(k) = \infty \quad (29)$$

which is in contradiction to (27). Thus, we have  $\bar{\kappa}^* = 0$ . In addition,  $V(e) = e^T(k)D^2e(k)$  and  $D^2$  is positive definite, therefore  $\mathbb{P}\{\lim_{k \rightarrow \infty} \zeta_i(k) = \varphi\} = 1$  holds for  $\forall i \in \mathcal{I}_S$ . ■

### 3.3 Dynamic Duty-cycle Scheduling Mechanism

In this subsection, a dynamic duty-cycle scheduling mechanism is proposed to improve the autonomy network estimation ability. The main idea is to awaken the sleeping node only when a virtual motion condition is satisfied, such that the energy consumption can be reduced and network lifetime is prolonged.

Assume that each SN can adjust its position. Construct a motion mechanism for each SN to maximize its sensing information. The sensing information is defined as a function of its current location  $p_i$ , sensor uncertainty  $R_i$ , the set of basis functions  $\psi$  of the environmental mode, and the model uncertainty as represented in the covariance matrix  $P_i$ . In addition, let  $A \in \mathbb{R}^2$  denote the bounded motion region monitored by relay assisted WSNs. Then, the quality of the current parameters estimation is inversely related to the cost function

$$J_i = \int_A \Psi(r)P_i\Psi^T(r)dA, \forall i \in \mathcal{I}_S \quad (30)$$

where  $A$  denotes the region,  $r \in A$ ,  $dA$  is a differential element of  $A$ . The cost  $J_i$  is the integral of the variance of model over the region of  $A$ .

With (30), we have the following gradient control law

$$\begin{aligned} f_i &= [f_i^X, f_i^Y]^T = -\frac{\partial J_i}{\partial p_i} \\ &= -\int_A \Psi(r)\frac{\partial P_i}{\partial p_i}\Psi^T(r)dA, \forall i \in \mathcal{I}_S \end{aligned} \quad (31)$$

where  $f_i^X$  and  $f_i^Y$  denote the gradient control laws in different directions.

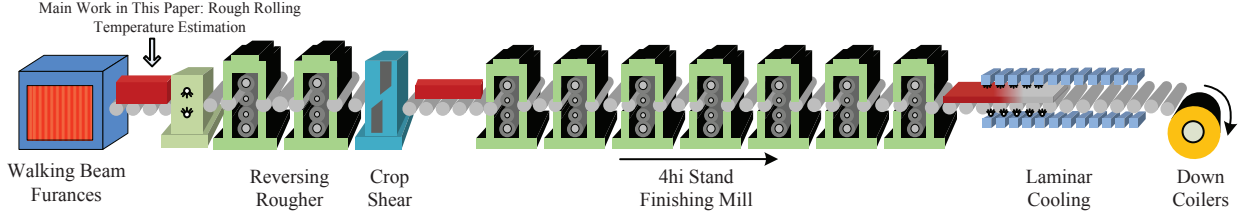


Fig. 2. A schematic view of hot rolling production line.

Under the control law (31), each SN moves according to the following dynamics

$$p_i(k+1) = p_i(k) + K f_i, \quad (32)$$

where  $K \in \mathbb{R}^+$  is a scalar gain.

In the mentioned design process, it is assumed that each SN can adjust its position. However, the sensors in industrial CPS are normally static, i.e., they cannot adjust their positions. How to make a virtual motion control to improve the estimation performance becomes a new problem. To solve this problem, we can adopt a dynamic duty-cycle scheduling mechanism shown in Algorithm 3, where  $\mu_{\#} \in \mathbb{R}^+$  is a decimal;  $\mathcal{I}_S^{\#}$  is defined as the sleeping SN set; the awake SN set is denoted as  $\mathcal{I}_S$ ; for an arbitrary SN  $i \in \mathcal{I}_S$ , its sleeping neighbors are defined as  $B_i = \mathcal{I}_S^{\#} \cap \mathcal{N}_i$ .

#### 4 APPLICATION TO SLAB TEMPERATURE ESTIMATION IN HOT ROLLING PROCESS MONITORING

In this section, the proposed distributed estimation approach is applied to estimate the slab temperature distribution in the monitoring for a hot rolling process which is a typical industrial CPS. The process consists of several stages to cast steel slabs into pieces, as shown in Fig. 2. During this process of roughing mill, the temperature plays an important role in the product quality. In fact, not only the calculation of the rolling force but also the metallurgical and mechanical properties of the products are critically influenced by the slab temperature [29]–[31]. Consequently, a precise estimation of the temperature distribution in the roughing mill contributes a lot to the quality of the final products.

Neglecting the variation of temperature in the transverse direction, the governing equation of heat transfer in the slab can be expressed as [29], [32]

$$\frac{\partial x(b, t)}{\partial t} = \frac{\lambda}{\rho c} \frac{\partial^2 x(b, t)}{\partial b^2} \quad (33)$$

where  $x(b, t)$  is the temperature of slab at time  $t$ ,  $b \in [-h/2, h/2]$  is the coordinate along the thickness  $h$ ,  $\rho(b)$ ,  $c(b)$  and  $\lambda(b)$  are the density, specific heat capacity and thermal conductivity of slab, respectively.

During rough rolling process, the heat loss on slab is mainly divided into two phases: 1) air cooling, and 2)

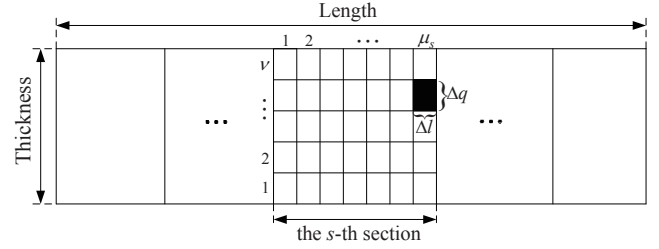


Fig. 3. The division of the rolled piece.

water cooling. In this paper, we focus on the temperature modeling in air cooling phase. Then the boundary conditions for (33) are given by

$$\begin{aligned} \lambda \frac{\partial x(b, t)}{\partial b} \Big|_{b=h/2} &= -\sigma_0 \varepsilon (x^4(h/2, t) - x_{\infty}^4), \\ \lambda \frac{\partial x(b, t)}{\partial b} \Big|_{b=-h/2} &= -\sigma_0 \varepsilon (x^4(-h/2, t) - x_{\infty}^4), \end{aligned}$$

where  $\sigma_0$  is the Stefan-Boltzmann constant,  $\varepsilon$  is the slab emissivity dependent on the temperature, and  $x_{\infty}$  is the environment temperature.

In practice, it is necessary to discretize the continuous model (33). To reduce the computational complexity, we divide the slab into  $\mu$  ( $\mu \geq 1$ ) strips. For each strip  $s$  ( $1 \leq s \leq \mu$ ), the length and thickness are divided into  $\mu_s$  and  $\nu$  lattices, respectively. Then  $\mu_s \nu$  grids can be obtained, as shown in Fig. 3. In each strip  $s$ , the grid, whose thickness and length numbers are  $i$  and  $j$  respectively, is labeled as  $x_{i,j}^s$ . Assuming that the time step is  $\Delta t$ , we obtain the following discretized model

$$\begin{aligned} x_{i,j}^s(k+1) &= x_{i,j}^s(k) + \frac{\Delta t \lambda_{i,j}^s}{\Delta b^2 \rho_{i,j}^s c_{i,j}^s} (x_{i+1,j}^s(k) - \\ & 2x_{i,j}^s(k) + x_{i-1,j}^s(k)), \end{aligned} \quad (34)$$

while at the surface

$$\begin{aligned} x_{\nu,j}^s(k+1) &= x_{\nu,j}^s(k) + \frac{2\Delta t \lambda_{\nu,j}^s}{\Delta b^2 \rho_{\nu,j}^s c_{\nu,j}^s} [x_{\nu-1,j}^s(k) - \\ & x_{\nu,j}^s(k) - \frac{\Delta b \sigma_0 \varepsilon}{\lambda_{\nu,j}^s} ((x_{\nu,j}^s(k))^4 - x_{\infty}^4)], \\ x_{1,j}^s(k+1) &= x_{1,j}^s(k) + \frac{2\Delta t \lambda_{1,j}^s}{\Delta b^2 \rho_{1,j}^s c_{1,j}^s} [x_{2,j}^s(k) - \\ & x_{1,j}^s(k) - \frac{\Delta z \sigma_0 \varepsilon}{\lambda_{1,j}^s} ((x_{1,j}^s(k))^4 - x_{\infty}^4)]. \end{aligned}$$

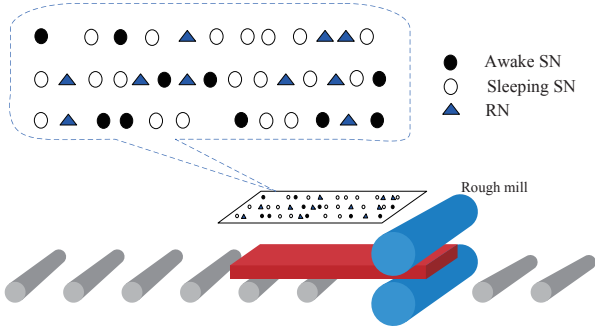


Fig. 4. Schematic illustration of the WSNs deployed before the entrance of the roughing mill.

Define  $x^s = [(x^{s(1)})^T, (x^{s(2)})^T, \dots, (x^{s(\mu_s)})^T]^T$ , where  $x^{s(j)} = [x_{1,j}^s, x_{2,j}^s, \dots, x_{\nu,j}^s]^T, \forall j = 1, 2, \dots, \mu_s$ . Stacking (34) into the following form

$$x^s(k+1) = A(k)x^s(k) - u(k), \quad (35)$$

with

$$\mathbb{Q} = \begin{bmatrix} -2 & 2 & 0 & \dots & 0 \\ 1 & 2 & 1 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & 1 & 2 & 1 \\ 0 & \dots & 0 & 2 & -2 \end{bmatrix},$$

$$\mathbb{P}_j = \begin{bmatrix} \beta(x_{1,j}^s)((x_{1,j}^s(k))^4 - x_\infty^4) \\ \mathbf{0} \\ \beta(x_{\nu,j}^s)((x_{\nu,j}^s(k))^4 - x_\infty^4) \end{bmatrix},$$

where  $A = I + \text{diag}\{\mathbb{D}_1\mathbb{Q}, \dots, \mathbb{D}_{\mu_s}\mathbb{Q}\}$ ,  $u = [\mathbb{P}_1^T, \dots, \mathbb{P}_{\mu_s}^T]^T$ ,  $\mathbb{D}_j = \text{diag}\{\alpha(x_{1,j}^s(k)), \dots, \alpha(x_{\nu,j}^s(k))\}$ , and  $\alpha(x_{1,j}^s(k)) = \Delta t \lambda_{\nu,j}^s / (\Delta b^2 \rho_{i,j}^s c_{i,j}^s)$ ,  $\beta(x_{p,j}^s) = \Delta t \sigma_0 \varepsilon / (\Delta b \rho_{p*,j}^s c_{p,j}^s)$ ,  $p_* = 1, \dots, \nu, j = 1, 2, \dots, \mu_s$ .

With the discrete model (35), we can easily obtain the internal temperatures of slab if the surface temperatures are all known. How to get the precise surface temperature distribution of slab? Obviously, increasing the number of sensors is not economical because slab is usually very long. To solve this problem, we model the temperature distribution of slab by using the measurements of limited number of sensors and relays. By this way, the surface temperature of slab can be estimated. For each section  $s$  ( $1 \leq s \leq \mu$ ), we divide the length and thickness into  $m$  and  $n$  subsystems respectively, where  $m < \mu_s$  and  $n < \nu$ . Then,  $mn$  grids can be obtained. Show each grid's temperature by the variable  $q_{i,j}^s$ , where the thickness and length numbers are  $i$  and  $j$ , respectively. Meanwhile, a model Gaussian radial basis function is constructed for the dynamics of  $q_{i,j}^s$

$$\psi_{i,j}^s(p) = \exp\left(-\frac{\|p - c_{i,j}^s\|^2}{b_*^2}\right),$$

where  $p \in \mathbb{R}^2$  is the position,  $b_* \in \mathbb{R}$  is the scalar parameter that defines the width for the radial unit, and  $c_{i,j}^s \in \mathbb{R}^2$  is the central location of  $q_{i,j}^s$ .

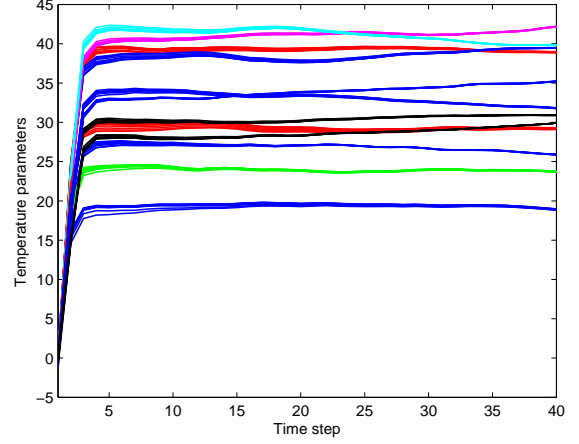


Fig. 5. The estimated temperature parameters with Algorithm 1.

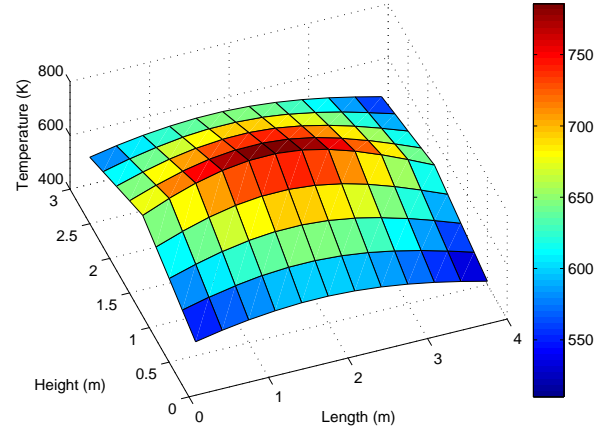


Fig. 6. Temperature distribution of slab in thickness direction.

The relay assisted WSNs are deployed over the top surface of the steel slab, as shown by Fig. 4. Next, we use the proposed estimation approach in Section 3 to estimate  $q^s$ . The parameters are set as follows:  $\sigma_0 = 5.67 \times 10^{-8} \text{W/m}^2\text{K}^4$ ,  $\Delta b = 1/3\text{m}$ ,  $\varepsilon = 0.85$ ,  $\lambda = 40\text{W/mK}$ ,  $x_\infty = 60 + 273\text{K}$ .

The hot steel slab studied in the simulation is  $h = 3\text{m}$  in thickness and  $l = 4\text{m}$  in length for each subsystem respectively. The other parameters are given as follows:  $\rho(k) = 0.6/(k+1)$ ,  $\mu_s = 12$ ,  $\nu = 9$ ,  $m = 4$ ,  $n = 3$ ,  $\mu_\# = 0.01\text{m}$ , and  $F = I_{12}$ . The 12 basis functions are used to describe the temperature parameters, with a standard deviation 5.5. The centers of the Gaussians are on the  $4 \times 3$  rectangle grids in the modeled region. The relay assisted WSNs are composed of awake 11 sensors including 8 SNs and 3 RNs. The sensor  $j$  can communicate with sensor  $i$  if  $r \leq 0.6$ . All disturbances are white Gaussian noises.

The initial values of the temperature estimate  $q^s$  are



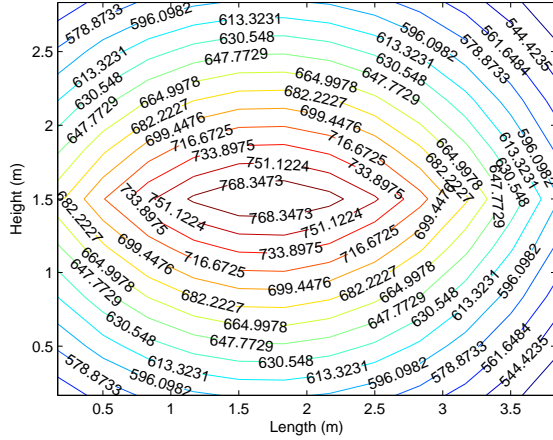


Fig. 7. Isotherm illustration of the temperature distribution.

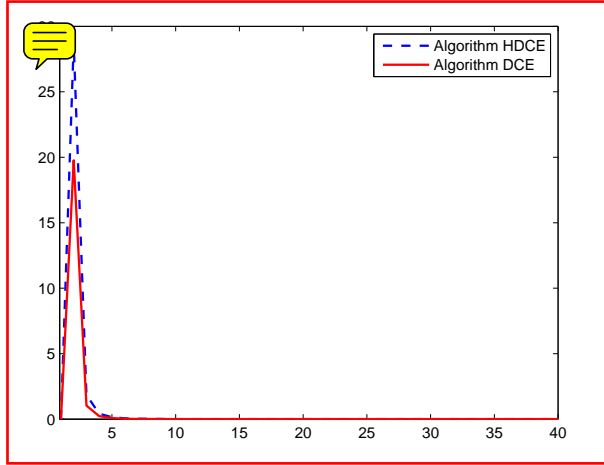


Fig. 8. Comparison result of the DCE algorithm with its homogeneous counterpart HDCE.

assumed to the zero. The initial position in length direction for these 8 SNs are designed as: 0.3m, 0.5m, 1.2m, 1.6m, 2.1m, 2.5m, 3.3m and 3.8m. Correspondingly, the measurements are given as:  $(320 + 273)K$ ,  $(300 + 273)K$ ,  $(280 + 273)K$ ,  $(270 + 273)K$ ,  $(330 + 273)K$ ,  $(240 + 273)K$ ,  $(350 + 273)K$ , and  $(230 + 273)K$ , respectively. With Algorithm 1, we can get a consensus estimation of the 12 temperature parameters, as shown in Fig. 5. With these measurements, we estimate the temperature distribution in the direction of thickness. The estimated temperature distribution is illustrated in Fig. 6. To show more clearly, we use isotherm in Fig. 7 to show the temperature distribution of slab in the direction of thickness.

Finally, we compare the proposed distributed consensus-based estimation (which is denoted by DCE) algorithm with its homogeneous counterpart, i.e. distributed algorithm (7) with all 11 sensors being SNs (which is denoted by HDCE). We only use the first row and the first column of the matrix  $\varphi$  to evaluate the performance, and the other entries of  $\varphi$  have the similar

comparison results. Thus, the mean squared error (MSE) is defined as  $e_{MSE}^{1 \times 1}(k) = \frac{1}{M} \sum_{i=1}^M \|\zeta_i^{1 \times 1} - \varphi^{1 \times 1}\|^2$ .

The comparison result is shown in Fig. 8. It is observed that although there are RNs, the performance of the DCE algorithm is better than the homogeneous algorithm HDCE. This reveals that in the presence of asymmetric communication for ubiquitous monitoring, more measurements may not lead to more accurate estimates, especially when there are root SNs. In fact, the variances of the estimation error of the unobserved variables increase at the root SNs.

## 5 CONCLUSION

The increasing applications of WSNs witness the fact that the cooperative effort of sensor and actuator nodes can provide powerful support for ubiquitous monitoring in industrial cyber-physical systems. In this paper, we have investigated the distributed parameter estimation problem for process monitoring over relay assisted WSNs composed of different kinds of SNs and RNs, where SNs take more responsibility of estimation and noise attenuating of the measurements, while RNs only play the role of data relaying and aggregation. For such a network, a Kalman filtering approach is first proposed to estimate the model parameters. Then, both a Tree-based broadcasting communication mechanism is presented to assist the fusion of global variables in the Kalman filter. Thus a distributed consensus-based estimation method is designed, such that the standard Kalman filter can be implemented in the distributive manner. Moreover, a dynamic duty-cycle scheduling mechanism is designed to get the virtual motion control of static sensors and relays. Therefore, the information perception capability is enhanced and estimation performance is improved. The proposed method has been applied to estimate the slab temperature in hot rolling process monitoring. It has been shown that the temperature distribution in the direction of thickness can be estimated effectively only with the measurement on the top surface of slab. Simulation results are given to validate the effectiveness of the proposed method.

In our future work, the virtual motion control of nodes can be designed to further expedite the information fusion such that the network-wide estimation capability can be enhanced with the cooperation of RNs and SNs.

## REFERENCES

- [1] X. Cao, P. Cheng, J. Chen, et al, "Cognitive radio based state estimation in cyber-physical systems," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 3, pp. 489-502, 2014.
- [2] X. Cao, P. Cheng, J. Chen, et al, "An online optimization approach for control and communication co-design in networked cyber-physical systems," *IEEE Transactions on Industrial Informatics*, vol. 9, no. 1, pp. 439-450, 2013.
- [3] J. Chen, X. Cao, P. Cheng, et al, "Distributed collaborative control for industrial automation with wireless sensor and actuator networks," *IEEE Transactions on Industrial Electronics*, vol. 57, no. 12, pp. 4219-4230, 2010.

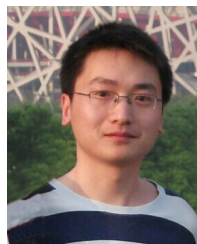
- [4] H. Zhu, S. Du, M. Li, et al, "Fairness-aware and privacy-preserving friend matching protocol in mobile social networks," *IEEE Transactions on Emerging Topics in Computing*, vol. 1, no. 1, pp. 192-200, 2013.
- [5] M. Dong, T. Kimata, K. Sugiura, et al, "Quality-of-Experience (QoE) in emerging mobile social networks," *IEICE Transactions*, vol. 97-D, no. 10, pp. 2606-2612, 2014.
- [6] V. Gungor, and G. Hancke, "Industrial wireless sensor networks: Challenges, design principles, and technical approaches," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 10, pp. 4258-4265, 2009.
- [7] K. Ota, M. Dong, Z. Cheng, et al, "ORACLE: Mobility control in wireless sensor and actor networks," *Computer Communications*, vol. 35, no. 9, pp. 1029-1037, 2012.
- [8] R. Olfati-Saber, and J. Shamma, "Consensus filters for sensor networks and distributed sensor fusion," in *Proceedings of IEEE Decision and Control Conference(CDC)*, Dec., 2005, pp. 6698-6703.
- [9] R. Carli, A. Chiuso, L. Schenato, and S. Zampieri, "Distributed Kalman filtering based on consensus strategies," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 4, pp. 622-633, 2008.
- [10] R. Rengaswamy, S. Narasimhan, and V. Kuppuraaj, "Receding-Horizon Nonlinear Kalman (RNK) Filter for State Estimation," *IEEE Transactions on Automatic Control*, vol. 58, no. 8, pp. 2054-2059, 2013.
- [11] N. Ilic, M. Stankovic, and S. Stankovic, "Adaptive consensus-based distributed target tracking in sensor networks with limited sensing range," *IEEE Transactions on Control Systems Technology*, vol. 22, no. 2, pp. 778-785, 2014.
- [12] R. Olfati-Saber, P. Jalalkamali, "Coupled distributed estimation and control for mobile sensor," *IEEE Transactions on Automatic Control*, vol. 57, no. 9, pp. 1-7, 2012.
- [13] S. Oh, Y. Xu, and J. Choi, "Explorative navigation of mobile sensor networks using sparse Gaussian processes," in *Proceedings of IEEE Decision and Control Conference(CDC)*, Dec., 2010, pp. 3851-3856.
- [14] K. Lynch, I. Schwartz, P. Yang, et al, "Decentralized environmental modeling by mobile sensor networks," *IEEE Transactions on Robotics*, vol. 24, no. 3, pp. 710-724, 2008.
- [15] M. Yarvis, N. Kushalnagar, H. Singh, et al, "Exploiting heterogeneity in sensor networks," in *Proceedings of IEEE INFORCOM*, March, 2005, pp. 878-890.
- [16] W. Li, F. Delicato, P. Pires, et al, "Efficient allocation of resources in multiple heterogeneous Wireless Sensor Networks," *Journal of Parallel and Distributed Computing*, vol. 74, no. 1, pp. 1775-1788, 2014.
- [17] S. Zhu, C. Chen, X. Ma, B. Yang, X. Guan, "Consensus based estimation over relay assisted sensor networks for situation monitoring," *IEEE Journal of Selected Topics in Signal Processing*, 2015.
- [18] S. Zhu, C. Chen, W. Li, et al, "Distributed optimal consensus filter for target tracking in heterogeneous sensor networks," *IEEE Transactions on Cybernetics*, vol. 43, no. 6, pp. 1963-1976, 2013.
- [19] S. Zhu, C. Chen, and X. Guan, "Sensor deployment for distributed estimation in heterogeneous wireless sensor networks," *Ad Hoc & Sensor Wireless Networks*, vol. 16, no. 4, pp. 297-322, 2012.
- [20] P. Gupta, P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388-404, 2000.
- [21] P. Jalalkamali, "Distributed tracking and information-driven control for mobile sensors networks," PhD Thesis, Dartmouth College, Hanover, 2012.
- [22] M. Jadaliha, Y. Xu, J. Choi, et al, "Gaussian process regression for sensor networks under localization uncertainty," *IEEE Transactions on Signal Processing*, vol. 61, no. 2, pp. 223-237, 2013.
- [23] X. Shen, "Green wireless communication networks," *IEEE Network*, vol. 27, no. 2, pp. 2-3, 2013.
- [24] R. Morris, E. Jolley, and A. Kodi, "Extending the performance and energy-efficiency of shared memory multicores with nanophotonic technology," *IEEE Transactions on Parallel and Distributed Systems*, vol. 25, no. 1, pp. 83-92, 2014.
- [25] H. Momeni, M. Sharifi, and M. Okhovvat, "Distributed assignment of real-time tasks in wireless sensor actor networks," *IEICE Electronics Express*, vol. 8, no. 7, pp. 429-435, 2011.
- [26] I. Jang, S. Yang, H. Yoon, et al, "EMBA: an efficient multihop broadcast protocol for asynchronous duty-cycled wireless sensor networks," *IEEE Transactions on Wireless Communications*, vol. 12, no. 4, pp. 1640-1650, 2013.
- [27] L. Yeh, and M. Pan, "Beacon scheduling for broadcast and convergcast in ZigBee wireless sensor networks," *Computer Communications*, vol. 38, no. 1, pp. 1-12, 2014.
- [28] H. Robbins, and D. Siegmund, "A convergence theorem for nonnegative almost supermartingales and some applications," *Optimizing Methods in Statistics*, J. S. Rustagi, Ed., Academic Press, New York, 1971.
- [29] S. Yildiz, J. Forbes, B. Huang, et al, "Dynamic modelling and simulation of a hot strip finishing mill," *Applied Mathematical Modelling*, vol. 33, no. 7, pp. 3208-3225, 2009.
- [30] C. Moon, and Y. Lee, "An approximate method for computing the temperature distribution over material thickness during hot flat rolling," *International Journal of Heat and Mass Transfer*, vol. 55, no. 1-3, pp. 310-315, 2012.
- [31] Y. Zheng, N. Li, and S. Li, "Hot-rolled strip laminar cooling process plant-wide temperature monitoring and control," *Control Engineering Practice*, vol. 21, no. 1, pp. 23-30, 2013.
- [32] S. Serajzadeh, A. Taheri, and F. Mucciardi, "Prediction of temperature distribution in the hot rolling of slabs," *Modelling and Simulation in Materials Science and Engineering*, vol. 10, no. 2, pp. 185-203, 2002.



**Cailian Chen** (S'03-M'06) received the B.Eng. and M.Eng. degrees in Automatic Control from Yanshan University, China in 2000 and 2002, respectively, and the Ph.D. degree in Control and Systems from City University of Hong Kong, Hong Kong SAR in 2006. She joined Department of Automation, Shanghai Jiao Tong University in 2008 as an Associate Professor. She is now a Full Professor.

Dr. Chen's research interests include Distributed Estimation and Control of Network Systems, Wireless Sensor and Actuator Network, Multi-agent Systems and Intelligent Control Systems. She has authored 2 research monographs and over 80 referred international journal and conference papers. She is the inventor of 20 patents. Dr. Chen received the "IEEE Transactions on Fuzzy Systems Outstanding Paper Award" in 2008. She was one of the First Prize Winners of Natural Science Award from The Ministry of Education of China in 2007. She was honored as "New Century Excellent Talents in University of China" and "Shanghai Rising Star" in 2013, "Shanghai Pujiang Scholar", "Chenguang Scholar" and "SMC Outstanding Young Staff of Shanghai Jiao Tong University" in 2009.

She is a Member of IEEE. She serves as an Associate Editor of Peer-to-peer Networking and Applications (Springer), ISRN Sensor Networks, and Scientific World Journal (Computer Science), and TPC member of several flagship conferences including IEEE Globecom, IEEE ICC and IEEE WCCI.



**Jing Yan** received the B.Eng. degree in Automation from Henan University, China in 2008, and Ph.D. degree in Control Theory and Control Engineering from Yanshan University, China in 2014. From June 2014 to December 2014, he was a Research Assistant at Key Laboratory of System Control and Information Processing, Ministry of Education of China. Currently, he is a Lecturer in Yanshan University, China.

Dr. Yan's research interests cover cyber-physical systems, multi-agent systems, and networked teleoperation systems. He has authored over 20 referred international journal and conference papers. He is the inventor of 2 patents. He was awarded the competitive National Graduate Scholarship from Ministry of Education of China in 2012. He received Excellence Paper Award from the National Doctoral Academic Forum of System Control and Information Processing in 2012. In 2013, he received Science and Technology Innovation Award from Yanshan University, China.



**Yiying Wang** Yiyin Wang received the B.S. degree in electrical engineering from Fudan University, Shanghai, China, in 2002, the M.S. degree (cum laude) in microelectronics from Delft University of Technology (TU Delft), The Netherlands, and Fudan University, China, in 2005, respectively, and the Ph.D. degree in electrical engineering from TU Delft, The Netherlands, in 2011. She is currently an Assistant Professor at the Department of Automation, Shanghai Jiao Tong University, China. Prior to that, she was a

Research Assistant at the Circuits and Systems group (CAS), TU Delft from 2006 to 2007. From February 2010 till July 2010, she visited Georgia Institute of Technology (Gatech), Atlanta, USA. She was a Postdoc Fellow at TU Delft and then Gatech from 2011 to 2013. Her research interests lie in the general area of signal processing for communications and networking. She currently focuses on tracking, localization, and synchronization for wireless sensor networks.



**Xinping Guan** (M'02–SM'04) received the Ph.D. degree in control and systems from Harbin Institute of Technology, Harbin, China in 1999. In 2007, he joined the Department of Automation, Shanghai Jiao Tong University, Shanghai, China, where he is currently a Distinguished University Professor, the Executive Deputy Dean of University Office of Research Management, and the Director of the Key Laboratory of Systems Control and Information Processing, Ministry of Education of China. Before that, he was the

Professor and Dean of Electrical Engineering, Yanshan University, China in 1998-2008.

Dr. Guan's current research interests include cyber-physical systems, multi-agent systems, wireless networking and applications in smart city and smart factory, and underwater sensor networks. He has authored and/or coauthored 4 research monographs, more than 180 papers in IEEE Transactions and other peer-reviewed journals, and numerous conference papers. As a Principal Investigator, he has finished/been working on many national key projects. He is the leader of the prestigious Innovative Research Team awarded by National Natural Science Foundation of China (NSFC). Dr. Guan is an Executive Committee Member of Chinese Automation Association Council and the Chinese Artificial Intelligence Association Council. He was on the editorial board of IEEE Trans. System, Man and Cybernetics–Part C and several Chinese journals. He received the First Prize of Natural Science Award from the Ministry of Education of China in 2006 and the Second Prize of the National Natural Science Award of China in 2008. He was a recipient of the "IEEE Transactions on Fuzzy Systems Outstanding Paper Award" in 2008. He is a "National Outstanding Youth" honored by NSFC, "Changjiang Scholar" by the Ministry of Education of China and "State-level Scholar" of "New Century Bai Qianwan Talent Program" of China.



**Ning Lu** (S'12) Ning Lu (S'12) received the B.Sc. and M.Sc. degrees from Tongji University, Shanghai, China, in 2007 and 2010, respectively. He is currently working toward the Ph.D. degree with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON, Canada. His current research interests include capacity and delay analysis, media access control, and routing protocol design for vehicular networks. Mr. Lu served as a Technical Program Committee Member for IEEE

2012 International Symposium on Personal, Indoor, and Mobile Radio Communications.



**Xian Yang** received her B. E. degree in automation from Yanshan University, China, in 2010. She is currently a Ph.D. candidate in navigation guidance and control in Yanshan University, China. Her research interest covers stability analysis of system with time-varying delay and bilateral teleoperation. E-mail: xyang@ysu.edu.cn